The paper deals with numerical and experimental solution of residual stresses, which occurs due to a contact loading. The first solved case corresponds to frictionless contact in an indent test accepted from literature. In the finite element analysis, three material options with isotropic, kinematic and combined hardening were tested. The model of Calloch and Marquis, which was implemented into the Ansys FE program by user subroutine, shows the best correlation with experiments. The experimental results were obtained by two experimental methods, namely by the Neutron diffraction method and by the Contour method, which is not well known in the Czech Republic. Hence, this paper includes also a brief description of the new destructive method. Both methods can lead to prediction of full field residual stresses distribution. Some difficulties from application of the contour method to own experiment in rolling contact fatigue domain are also discussed.

**Keywords**

residual stress, plasticity, contour method, FEM, rolling contact fatigue

### 2. Numerical solution

Recently, various numerical methods are used for residual stress analysis. The most applied methods in contact mechanics are the finite element method [Wriggers 2006] and the boundary element method [Beer 2001]. Material model incorporated in such analysis has the main influence on accuracy of a numerical analysis in the case of residual stresses induced by contact loading. However, classical plasticity models with a nonlinear hardening rule for metals included now in commercial computational software are not able to describe the nonproportional hardening effect that appears for a lot of metallic materials.

A simulation of an indent test will be described to show the problem better. In the solved case, residual stresses due to plastic deformation of indented specimen from 316L steel are known through experimental works of Pagliaro et al [Pagliaro 2007]. In the paper, particulars of experimental setup are presented in detail. A 60-mm diameter and 10-mm thick disk of 316L stainless steel was plastically compressed through the thickness with a 15 mm diameter flat indenter in the center of the disk. A schema of the contact task is clear from Fig.1.

![Figure 1. Scheme of the experiment taken from [Pagliaro 2007]](image)

Initially, for own simulation, the axisymmetric FE model of the indenter and the specimen were built with 8132 PLANE42 elements defined by 8408 nodes and 15000 PLANE42 elements defined by 15351 nodes respectively. Appropriate symmetric and antisymmetric boundary conditions and a displacement of -0.12 mm were applied at the top of the indenter, see Fig. 2. Then, the augmented Lagrangian frictionless surface-to-surface contact algorithm was utilized to treat contact conditions. The indenter was modelled by one single superelement [Halama 2008], because of linear elastic material assumption (A2 tool steel, yield stress about 1300MPa, Young’s modulus E=204000MPa and Poisson’s ratio v=0.3). Multilinear kinematic hardening, multilinear isotropic hardening and nonlinear combined hardening options were assumed for the stress-strain behaviour characterisation of the specimen model. User programmable features (UPFs) approach was applied to implement robust plasticity model of Calloch and Marquis [Calloch 1999] with combined hardening rule into the FE code. Its brief description is given by Table 1. The same values of material parameters as in the paper [Calloch 1999] were used in the performed analysis. The multilinear isotropic and multilinear kinematic hardening models were calibrated using monotonic strain curve only.

![Von Mises yield criterion](image)

Voš Mises yield criterion $\sigma = \frac{1}{\sqrt{2(1+\nu)}} \left( \sigma_x - 2 \tau_{yz} \right)$  where $\sigma$ is the deviatoric part of stress tensor $\sigma$ as is the deviatoric part of back-stress vector $\sigma$, $R$ is the isotropic scalar variable and $\sigma_0$ is the initial size of the yield surface.

The Armstrong-Frederick kinematic hardening rule $\dot{\sigma} = \frac{2}{3} (\dot{\varepsilon}_p - \dot{\varepsilon}_0) \dot{\varepsilon}_p$  where $\dot{\varepsilon}_p$ is the plastic strain rate and $\dot{\varepsilon}_0$ is the initial strain rate.

The Bauschinger isotropic hardening rule $R = b(2 - R) p$, $Q = D_0 b(\dot{\varepsilon}_p - \dot{\varepsilon}_0)$, $A = \frac{1}{2} \left( \frac{\sigma}{\sigma_0} \right)^m$.

<table>
<thead>
<tr>
<th>Material parameters</th>
<th>$\sigma_0$=160MPa</th>
<th>$c_1=1800$MPa</th>
<th>$\nu=0.32$</th>
<th>$\bar{Q}_0=2410$MPa</th>
<th>$\bar{Q}_1=2334$MPa</th>
<th>$\phi_0=0.0048$</th>
<th>$\phi_0=0.0017$</th>
<th>$\alpha=15$</th>
<th>$T_0=0.05$</th>
<th>$r=6.26$</th>
<th>$\gamma=0.01$</th>
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Table 1. Description of the plasticity model with combined hardening rule

At the end of unloading process, the residual stresses remained in the specimen. The comparison of numerical and experimental results of the hoop residual stresses on symmetry plane (see appropriate symmetric boundary conditions at the Fig.2 and the cross-section of the specimen are shown in the graph printed as the Fig.3.

The simple plasticity options with isotropic and kinematic hardening give less accurate results then the model of Calloch and Marquis. It was recognized by many authors, that multilinear kinematic hardening model of Besseling and other models included in Ansys 11 cannot describe stress-strain response of 316L steel correctly mainly under nonproportional loading.
3. Experimental solution

Mechanical methods, mainly the hole-drilling method and the slitting method [Cheng 2007], or non-destructive methods, especially the neutron diffraction method and the X-ray diffraction method [Kolarik 2009], are commonly used for near surface residual stress measurement. During two last decades, hybrid methods have been developed in the experimental mechanics area, which mostly combine the best of an experimental and a numerical approach. The new such method was developed by Michal B. Prime [Prime 2001].

3.1 Contour method

This destructive method is suitable for measuring a cross-sectional map of residual stresses. Basic idea of so-called contour method is quite simple, see Fig.4. The situation A shows the body with original residual stress distribution. After cutting in half along a flat plane a relaxation of residual stresses caused the body to deform (B). Therefore, deviation of the cut plane is measured. Next step involves forcing back the body to its original configuration along the new free boundary (C). Assuming that the relaxation process was elastic, the body has been return to its original stress state (A=B+C). Described principle corresponds to a classical superposition principle.

Practically, the ideal method for making the cut has been proven to be Wire electric discharge machining (EDM), because the cutting is non-contact and would not cause any plastic deformation. Surface contour measuring can be realised for example by a coordinate measuring machine [Prime 2001] using touch or laser probe. More precise results of the method are obtained after data smoothing. As the last step, a finite element analysis is suitable for stress calculation. The average surface profile from part I and II is usually used to apply boundary condition, displacement normal to the cut plane, on a FE model.

3.2 Rolling contact residual stresses

The contact fatigue phenomenon under rolling condition has been investigated theoretically and experimentally at the VŠB-Technical university of Ostrava [Halama 2005]. There has been developed the unique test machine TUORS (Technical University of Ostrava – Rolling Sliding wear testing machine) during five last years, which makes possible simulating rail/wheel contact under different test conditions. The usual ratio of specimen diameters is approximately 2.6. Thus, the diameter of the driver is bigger than that of the follower, as could be seen at the Fig.5.

The resistance to wear and fatigue crack initiation and propagation of railway wheelset materials is mainly investigated by the TUORS testing machine. The testing device was described elsewhere [Halama 2009]. The residual stresses induced by repeated contact loading are usually measured by hole-drilling method [Macura 2007]. By this way, it is possible to get the residual stress distribution in subsurface layer. The measurements are often supplied by application of x-ray diffraction method to surface residual stress measurements [Kolarik 2008]. In the case of point contact tests the residual stresses induced by contact loading are strongly changed in lateral direction. Performing measurements in more places is very time consuming and expensive. If the contour method would be successful in this case, it will be used in advance.

As a verification case was chosen the point contact test, where the testing pulley (testing surface of the specimen – the sphere with diameter 82mm, CSN 11523 steel) was rolled on the testing rim (cylinder with diameter 215.4mm, CSN 15260.6 steel) under free rolling condition. The applied load leads to the maximum of Hertzian pres-

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**Figure 2.** Applied boundary conditions (B.C.)

**Figure 3.** Residual stress distribution on symmetry plane (left) and at the cross-section (right), all experimental data from [Pagliaro 2007]

**Figure 4.** Principle of the contour method

**Figure 5.** A photo from rolling contact test realisation

**Figure 6.** Contact area shape changes during rolling contact test
sure 1833MPa. The test was always interrupted after certain number of cycles. Then, size and shape of contact area was gained by the pressure sensitive film Pressurex. The contact area change with absorbed number of cycles is shown in the Fig. 6.

Recently, pressure sensitive films with resolution ability up to 300MPa are made, therefore the change of red color intensity can be seen only near edges of contact areas. It is clear, that the shape of contact area is changing from elliptical to near rectangular with increase of rolling distance. After 60000 rolling cycles, the similar pressure distribution as in the line rolling contact case can be expected (so-called Kunert type distribution, see [Kapoor 1992]). The diameter of the testing pulley was also measured during each test suspension. The diagram characterizing the surface flow is placed in the Fig.7.

The cut of the sample was made using EDM with a 150 µm diameter brass wire with the part submerged in temperature-controlled deionized water. According to the author of the contour method [Prime 2001], very slow cutting mode was applied to minimize cutting induced stresses. The time needed to cut the sample was approximately 2.5 hours.

The optical profilometer MicroProf FRT was used to scan the profile of cutting surface. The device allows scanning with a vertical resolution of up to 3 nanometers. Sampling was carried out with step size of 5µm. Time of scanning for one cutting area was approximately 12 hours and the corresponding data file in ASCII format has approximately 285 megabytes. Subsequent data processing was performed by LabVIEW software. Topography of cutting surface obtained by optical profilometer is evident from Fig. 9.

Note that the axis at the graph have distinctly different range. As can be seen from the Fig.9, peaks arise after the EDM machining near the edge of the cutting tip. These are probably caused by large temperature gradient at the interface during cutting. It was recommended in [Pagliaro 2007] to cut two samples, the loaded specimen and the blank one. Therefore, during the following data reduction and analysis process, it can be considered, which peaks may be filtered.

4. Conclusions
The main scope of this work was comparison of a numerical solution and an experimental solution of residual stresses induced by a contact loading. In service, such a loading is often repeated, for example in Wheel/Rail systems or bearings. The numerical analysis of the simple indent test taken from literature [Pagliaro 2007] indicates, that it is not sufficient to use only monotonic strain curve for material model calibration. The cyclic plasticity model of Calloch and Marquis, implemented into the Ansys program, gives better results than the purely isotropic and kinematic multilinear hardening models. On the other hands, results from the finite element analysis show the need of next search for a more accurate material model for stress-strain behaviour description of the deemed stainless steel 316L. Probably, it is necessary to include a yield surface distortion theory in the plasticity laws to get results closer to the experimental results [Benallal 1989].

The article describes also a new experimental method for measuring the plane distribution of residual stress, called the contour method. This method combines experimental approach with the numerical one and requires cutting the sample in the selected plane. The first two steps of the contour method were conducted for a rolling contact test evaluation to show some pitfalls of the method. The observed profile of the cut surface shows the problematic evaluation of residual stresses on the surface and to a depth of approximately 0.1 mm. Thus, the hole-drilling method and the x-ray diffraction method have been the most useful methods yet for residual stress measurement in rolling contact fatigue experimental research [Macura 2008]. For other applications the method may be appropriate and could fill a gap in the field of experimental mechanics for determining residual stress in greater depth of material, where traditional methods often fail.

Acknowledgements
This work was supported by MSM6198910027 project of The Ministry of Education, Youth and Sports.


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