DESIGN OF METHOD FOR NONLINEAR SHEAR MODULUS MEASURING IN GEL-LIKE MEDIUM BY APPLYING AN ADDITIONAL STATIC STRESS TO AN ACOUSTIC RESONATOR

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Measurements of the nonlinear parameter of a gel-like medium were carried out in an acoustical resonator fixed without slipping between two solid-state boundaries. A sample of thickness *L* is fixed on an oscillating plate (x = 0). The other plate of finite mass on the free surface of the sample (x = L) moves together with this surface. By changing the mass of the plate (x = L), it is possible to achieve additional static deformation of the resonator up to 65%. The dynamic method assumes measuring resonance curves at various static deformations. Nonlinear properties appear at deformations of more than 20%. The nonlinear parameter and shear modulus measured dynamically were compared to static measurements where the dependence becomes nonlinear at strains greater than 30%. The static values of the shear modulus and the nonlinear parameter correspond to the values obtained in the dynamic method within an error.

KEYWORDS

acoustical resonator, gel-like media, shear moduli, cubic nonlinearity, relaxation

1 INTRODUCTION

Establishing the relationship between rheological parameters and characteristics of shear waves in viscoelastic media has become a very important problem. From a fundamental point of view, establishing the relationship between rheological parameters and the characteristics of shear waves in gel-like media will lead to the further development of ideas about the viscoelastic properties of such media, and will allow a more detailed study of the dependence of rheological parameters on the method of action on the medium. Ultimately, refined physical models of gel-like media will appear [Krit 2020]. The practical value lies in the application of the results obtained for modeling viscoelastic media and creating high-precision methods [Bozek 2016] for measuring their elastic properties. The relevance of a detailed study of viscoelastic media is due to the fact they are used as a matrix for tissue engineering.

The measurement of the viscoelastic properties of viscoelastic media and the creation of adequate models of these materials

has become relevant since its solutions can be helpful for the development of tissue engineering methods. The measurements of the shear modulus and shear viscosity of materials in a wide frequency range make it possible to predict the response of the certain material to pulsed mechanical effects correctly [Krenicky 2021, Kaminski 2022, Szweda 2022]. The fundamental importance of this research lies in the creation of physical models that will lead to the further development of ideas about the viscoelastic properties of gel-like media.

MATERIAL AND METHODS

We study gel-like media – solid incompressible media with the shear moduli varying from a few kilopascals to several megapascals. The shear modulus is uniquely related to the shear wave velocity and together with the lambda parameter, determines the longitudinal wave velocity [Asfandiyarov 2021, Sarvazyan 2010]. Along with the shear modulus, Young's modulus is also a convenient characteristic of elasticity. The bulk modulus of gel-like media *K* exceeds the shear modulus by several orders of magnitude. Therefore, shear modulus μ is much less than *K*. In this approximation, the lambda parameter is approximately equal to the bulk modulus. Young's modulus is three times greater than shear modulus, and Poisson's ratio is equal to one half.

Due to the fact that the shear modulus in gel-like media varies over a much wider range than the bulk modulus, its changes are registered easily. Thus, shear waves in gel-like media become a highly accurate investigation tool. We used the method of the acoustical resonator which is a convenient tool for excitation of the shear waves in gel-like media.

1.1 Estimation of nonlinear shear elastic modulus in the gellike medium

Consider an acoustical resonator in the form of rectangular parallelepiped fixed without slipping between two solid-state boundaries (Fig. 1). Vibrating boundary (x = 0) is forced with the harmonic function. The boundary with variable mass is forced by the elastic layer (x = L). If one of these plates has coordinate x = 0, and the distance between the plates (the thickness of the resonator) is L, another plate should have coordinate x = L. Shear waves in the resonator excited by the displacement across the x axis propagate along the x axis.



Figure 1. Gel-like resonator between two solid-state boundaries (elastic layer)

1.2 Static measurements of nonlinear shear elastic properties

Prior to the investigation of the shear waves in the resonator we studied how the shear stress depends on the relative deformation. We constructed the experimental setup shown on Fig. 2. The resonator was made of plastisol. Layer was solidified between two parallel wooden plates. The plate at x = 0 was

immobilized, whereas the opposite plate was loaded with a variable mass connected through the pulley.



Figure 2. The experimental setup for static measurements of the shear stress on the relative deformation

The measured shear stress σ on the relative deformation ε is shown on Fig. 3. The dependence is nonlinear and is approximated well with the cubic parabola [Andreev 2011b, Krit 2014].



Figure 3. Measured shear stress on the relative deformation

Thus, the dependence of the effective shear modulus on the relative deformation (Fig. 4) found as $\mu_{eff} = \sigma / \varepsilon$ is well approximated with a square parabola equation: $\mu_{eff} = \mu_0 (1 + \delta \varepsilon^2)$. It becomes evident then that the effective shear modulus of the resonator material increases i.e., the material changes its elastic properties as the deformation increases. The approximation with the square parabola gives the values $\mu = 13.46 \pm 0.13$ kPa, $\delta = 0.60 \pm 0.07$.



Figure 4. Measured effective shear modulus on the relative deformation

1.3 Estimation of nonlinear shear elastic properties applying vibrational methods

Finite amplitude vibrations

As the increase of the shear modulus leads to changes of the elastic properties, the material response to the shear waves depends on their amplitude. These effects were carefully studied in [Andreev 2011b]. According to the law of motion, every shear displacement *u* of the resonator material causes the shear stress σ inside the material. We have shown before [Andreev 2011a] that if shear displacement appears along the side of the resonator which is at least 4 times longer than the distance between the solid-state boundaries *L*, a one-dimensional model of thin resonator is appropriate to shear waves description in such an elastic layer.

To represent the connection between shear stress and shear displacement we used the mathematical model [Andreev 2011b] with a single relaxation time τ , based on the equations

$$\frac{\partial v}{\partial t} = \frac{1}{\rho} \frac{\partial \sigma}{\partial t} \tag{1}$$

where \boldsymbol{v} is the shear wave velocity, $\boldsymbol{\rho}$ is the density of the resonator, and

$$\sigma = \sigma_{\infty} + \sigma' \tag{2}$$

In (2):

$$\frac{\partial \sigma'}{\partial t} + \frac{\sigma'}{\tau} = \frac{\eta}{\tau} \frac{\partial \varepsilon}{\partial t}$$
(3)

$$\frac{\partial \sigma_{\infty}}{\partial t} = \mu_0 (1 + 3\beta \varepsilon^2) \frac{\partial v}{\partial y} \tag{4}$$

where $\mu = \mu_0 (1 + 3\beta\varepsilon^2)$ is the shear modulus and η is the shear viscosity of the resonator material. Vibrating boundary (x = 0) is forced with the harmonic function. Thus, its acceleration w is represented with the harmonic function too:

$$w(t)|_{x=0} = W_0 \cos \omega t \tag{5}$$

where W_0 is the amplitude and ω is the cyclic frequency. Another boundary is forced by the elastic layer (x = L). Its law of motion is the following:

$$\left. \left(\frac{M}{s} \frac{\partial v}{\partial t} + \sigma \right) \right|_{x=L} = 0 \tag{6}$$

2 METHODS AND MATERIALS

In [Andreev 2011b] we have shown that the system of equations (1)-(2) together with the boundary conditions (5)-(6) has a frequency dependent solution $|W_L/W_0|$, where W_L is the amplitude of the acceleration of the boundary at x = L. This solution is represented for the value $W_0 = 1 \text{ m/s}^2$ on Fig. 5. Measured ratio is shown by dots and approximated by the red line for the resonator with μ = 13.46 kPa, θ = 0.60, η = 4.7 Pa·s and relaxation time τ = 0.7 ms. Blue line shows the same solution in case η = 2.4 Pa·s. Green dash line shows the same solution in case τ = 0. The relaxation of the material turns into the frequency dependence of its shear modulus and shear viscosity. All the solutions represented on Fig. 5 have peaks at certain frequencies. In the resonator, the frequencies where these peaks occur (resonance frequencies) depend on the mass of the plate on the boundary x = L. Inset on Fig. 5 shows how the resonance frequencies depend on the plate mass. The slight increase of the plate mass leads to decrease of the resonance frequencies. The resonance peak with the lowest frequency (first resonance) is the highest. Thus, it is more convenient to study nonlinear effects in the region of the first resonance frequency.



Figure 5. The ratio of the acceleration amplitudes $|W_L/W_0|$ for the value $W_0 = 1 \text{ m/s}^2$

It was shown in [Andreev 2011b] that at the certain acceleration amplitude of the boundary at x = 0 the velocity amplitude of the resonator boundary at x = L constitutes about 40% of the shear wave velocity. At these particle velocities in a resonator, nonlinear effects manifest themselves. The resonance becomes asymmetrical, and the resonance frequency increases. The acceleration gain factor also increases in comparison with the linear case. As the amplitude W_0 grows more, the resonance curves obtained for increasing frequency and its decrease do not coincide; i.e., a bistable region arises. The bistable region widens as the oscillation amplitude in a resonator grows.

Small amplitude vibrations under severe static load

The excitation of the finite amplitude vibrations is complicated [Abramov 2015, Bozek 2021, Nikitin 2020, Peterka 2020]. Thus, we suggested an algorithm for measurement of the nonlinear shear modulus in a gel-like medium applying small vibrations to the resonator. As it was already discussed, the effective shear modulus of the resonator material increases with the increase of the deformation. So, the algorithm is based on the nonlinear change of the effective shear modulus under the static load together with the linear oscillations applied to the resonator. The linear model was described in [Andreev 2010] by the equations

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} \tag{7}$$

$$\sigma = \mu \frac{\partial u}{\partial x} + \eta \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial t} \right) \tag{8}$$

followed by the boundary conditions (5)-(6).

The frequency dependence of the ratio near the first resonance is shown on Fig. 6 by the line. Calculated resonance curve is shown by line, measured resonance curve is shown by dots. Since the acceleration amplitudes are small enough, the velocity of the oscillations of the boundary at x = L constitutes 10% and less of the shear wave velocity. The shear stress depends linearly on the shear displacement values corresponding to the mentioned amplitudes. Thus, the shear modulus remains constant. However, the value of the shear modulus varies under the static deformation, as it was shown on Fig. 4.



Figure 6. The frequency dependence of the ratio near the first resonance for the resonator without static load excited with the oscillations of amplitude $W_0 = 1 \text{ m/s}^2$

Due to this fact, we suggest applying various static shear loads to the resonator changing the operating point on the relationship between shear deformation and shear stress caused by it. Linear oscillations near a certain operating point do not affect the measured value of the shear modulus, whereas the measured shear modulus itself changes nonlinearly according to the cubic law [Krit 2021].

3 DISCUSSION

The described shear wave excitation method is illustrated on Fig. 7.



Figure 7. The experimental setup for excitation of shear waves with a small amplitude under severe static load: (1) – resonator; (2) – the excited boundary; (3) – plate of a finite mass; (4) – Brüel&Kjær Type 4810 shaker; (5) – signal amplifier MMF LV 103; (6) – function generator Rigol DG1062Z; (7) – miniature piezoelectric charge accelerometer Brüel&Kjær Type 4374; (8) – charge amplifier Brüel&Kjær Type 2635; (9) – digital oscilloscope GaGe CompuScope Express 4444; (10) – the rods; (11) – the weights

The resonator (1) is placed vertically so that one of its boundaries (2) is excited by the miniature shaker Brüel&Kjær Type 4810, while the other moves the plate of a finite mass (3). The miniature shaker Brüel&Kjær Type 4810 (4) is connected to the signal amplifier MMF LV 103 (5) which is powered from the function generator Rigol DG1062Z (6). The acceleration of the plate (3) is measured with a miniature piezoelectric charge accelerometer Brüel&Kjær Type 4374 (7). Signal from the accelerometer is amplified at the corresponding charge amplifier Brüel&Kjær Type 2635 (8) and received with the corresponding

channel of digital oscilloscope GaGe CompuScope Express 4444 (9) in order to be compared to the acceleration at boundary (2). Rods (10) are attached to the plate (3) for the weights (11) to be placed in order to create a static deformation of the resonator. Measured shear stress τ on the relative deformation ε is shown by dots on Fig. 8. Oscillations at the operating points are shown schematically within the region of linear (ε = 0.075) and nonlinear (ε = 0.423) deformations.



Figure 8. Measured shear stress τ on the relative deformation ϵ (dots) approximated by the cubic parabola (blue line) and with the straight red line

Dots are approximated by the cubic parabola (blue line). The straight red line approximates the measured dots in the area of small deformations, where the relationship is linear. In the experiment, we used several operating points. Two of them correspond to the relative deformation values ε = 0.075 and ε = 0.423. These points are shown on the plot and the oscillations about them are shown schematically within the region of deformations. During the oscillations about the point shown at ε = 0.075 the shear stress depends on the shear deformation linearly. The deformations do not exceed the values that correspond to the dots lying on the straight red line. As the static load grows, the deformations take on the values that do not correspond to the red line. However, the spread of the values is still small, since the amplitudes of the displacement correspond to the linear case with the higher slope. Thus, as the shear modulus grows together with the slope on Fig. 8, the resonance frequency becomes higher. The measured frequency near the operating point provided us with the effective shear modulus making it possible to measure both linear shear modulus μ = 15.4 \pm 1.1 kPa and nonlinear coefficient θ = 0.53 \pm 0.06.

4 CONCLUSION

We suggested a novel approach based on measuring resonance curves at low amplitudes with an applied static shear stress that creates nonlinear deformation for measurement of the nonlinear viscoelastic properties in gel-like media. It is based on the combination of static and dynamic methods. Static load is used to make the studied media show their nonlinear behavior, whereas the resonator method lets us measure the frequency dependence of the shear elastic modulus with a high precision. The effective shear modulus of the material for a given resonator load depends on the resonance frequency. The standing shear wave excitation method can be implemented into the industrial devices for non-destructive testing [Jakubovicova 2017, Kuric 2021, Macko 2017]. An extra load attached to the resonator changes the resonance frequencies in two ways. An extra mass itself reduces the frequencies [Krit 2015a,b, Smeringaiova 2021], whereas the nonlinear effects due to the deformation under the load leads to the increase of the frequencies. Both of these effects are taken into account in our model. Shear displacement appears along the side of the resonator which is more than 4 times longer than the distance between the solid-state boundaries. Therefore, a one-dimensional model of a thin layer is appropriate. In the dynamic method, nonlinear properties appear at strains of more than 20%. Approximation by a cubic polynomial makes it possible to determine the nonlinear parameter. Practical applications are also possible in the area of frictional stress as reported by the authors [Frankovsky 2016, Krawiec 2017, Medvecka-Benova 2015, Mikova 2022].

It was found that when measured by the dynamic method, the nonlinear properties of the material appear at lower strains than when measured by the static method. At a static deformation of more than 20%, the nonlinearity of the shear modulus of plastisol appears, which leads to an increase in the resonance frequency in comparison with the absolutely linear case [Andreev 2010]. This effect is common for plastisol [Andreev 2011b].

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