

# INFLUENCE OF TESTING METHODS ON THE FINAL VALUES OF THE MODULUS OF ELASTICITY E

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In the area of elastic deformation there is linear dependence between the applied stress and final deformation, which is described by the Hooke's law. Thus stress and strain are proportional to each other and constant of proportionality is termed as modulus of elasticity (even. Young's modulus). So this material constant characterizes the elastic deformation behavior of the given material and its magnitude represents the very important material constant in all industrial branches and mainly determines the dimensions for the given parts. That is why it's truly very important to precisely measure its values for different materials. Because of this were in this article used two methods for measurement Young's modulus E (MPa) – static tensile test and three-point bend test. Main aim was to obtain statistical evaluation about influence of these testing methods on the final values of modulus of elasticity.

## KEYWORDS

modulus of elasticity, elastic deformation, linear fitting, material tests, slope of line, f-test, student's t-test

## 1 INTRODUCTION

There are a lot of different methods how to determine the magnitude of Young's modulus E (MPa). The most common method arises from static tensile test where Young's modulus is taken as slope of engineering stress-strain curve [Ashby 2007]. However, elastic deformation represents a very small amount from the total deformation, so there is truly necessary to have an extensometer of high accuracy [Pöhlandt 1998]. As another possibility (and much more cheaper) there is 3-point bend test. In this case is Young's modulus calculated from deflection of specimen. This method is quite fast, but very sensitive on the initial bending of specimen. Acc. to standard (CSN EN ISO 7438) is this problem solved by adjustment of required distance between supports L (mm). However, it seems that especially for thin materials this is not enough. That's why for the experimental part there were chosen two materials with different thicknesses - aluminium alloy A6016 (t = 1,520 mm) and deep-drawing material DC06 (t = 0,710 mm).

The chosen materials represent quite common materials which are used in the car-body design [Davies 2003]. Aluminium alloy A6016 is alloyed by magnesium and silicon [Polmear2006]. Wide range of values of Young's modulus can be theoretically explained by differences in strength of the interatomic forces between adjacent atoms or ions [Hertzberg 1996]. However, the major aim of this paper was to carry out hypothesis testing if the selection of testing method (static tensile test and 3-point bend test) has/hasn't significant influence on the final values of Young's modulus E (MPa) and how these results can be influenced by material thickness. At static tensile test was used high-accuracy extensometer so this test was a reference one.

## 2 METHODOLOGICAL BASES AND EXPERIMENTAL PART

Thus there were two testing materials (A6016 and DC06) of different thicknesses and two testing methods (static tensile test and 3-point bend test). The basic mechanical properties of tested materials are shown below. Subsequent performance of testing methods and results (including hypothesis testing) is described in the following chapters.

### 2.1 Aluminium alloy A6016

Tab. 1 illustrates the basic mechanical properties of aluminium alloy A6016 and fig. 1 shows engineering stress-strain curve. Width of specimen was b = 20,18 mm and rolling direction 0°.

Material	Proof Yield Str. $R_{p0,2}$ (MPa)	Ultimate Strength $R_m$ (MPa)	Uniform Ductility $A_g$ (%)	Total Ductility $A_{50mm}$ (%)
A6016	106,9	196,4	24,27	28,90

Table 1. Mechanical properties of A6016

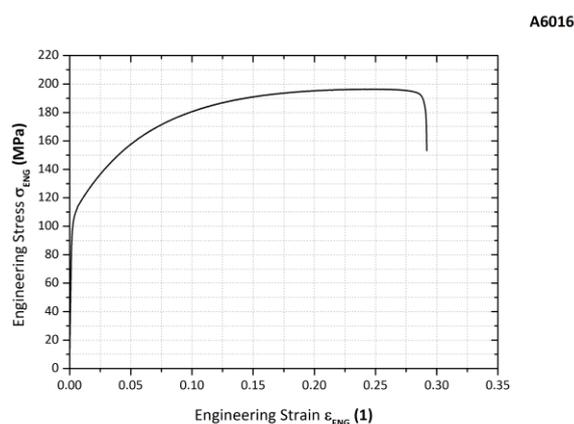


Figure 1. Engineering stress-strain curve for A6016

### 2.2 Deep-drawing material DC06

Tab. 2 illustrates the mechanical properties of deep-drawing material DC06 and fig. 2 shows engineering stress-strain curve. Width of specimen was b = 20,18 mm and rolling direction 0°.

Material	Proof Yield Str. $R_{p0,2}$ (MPa)	Ultimate Strength $R_m$ (MPa)	Uniform Ductility $A_g$ (%)	Total Ductility $A_{50mm}$ (%)
DC06	148,2	307,0	25,30	43,58

Table 2. Mechanical properties of DC06

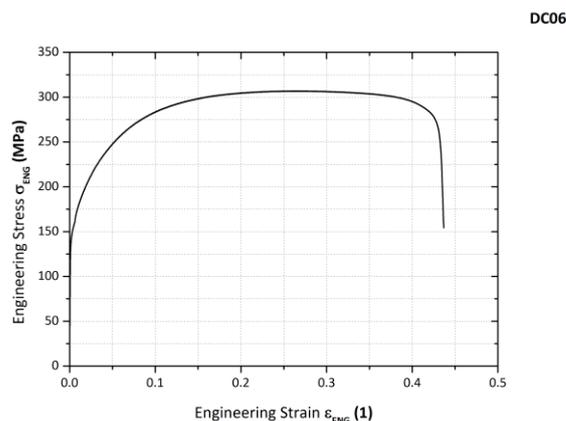


Figure 2. Engineering stress-strain curve for DC06

### 3 ALUMINIUM ALLOY A6016

As there was already written in the first chapter, two different tests were used to determine Young's modulus of tested materials, static tensile test and 3-point bend test, respectively. On this page are shortly described these testing methods and their evaluation for aluminium alloy A6016. In both cases were tested 10 specimens and statistical evaluation of results is then graphically shown in fig. 5 (by mean and standard deviation).

#### 3.1 Evaluation of the static tensile test (STT)

Determination of the Young's modulus E (MPa) by means of the static tensile test was carried out on the device TiraTEST 2300. To achieve the very high accuracy of this measurement, as an extensometer was used model 3452-010M-025-ST from the company Epsilon. Its travel is ±2,5 mm (±25%). Young's modulus E (MPa) was subsequently directly taken as the slope of  $\sigma_{ENG} - \epsilon_{ENG}$  curve. The used y-range was 10 MPa to 30 MPa. In fig. 3 is shown engineering stress-strain curve and linear fitting to determine slope (here E = 68002 MPa) for the 1<sup>st</sup> specimen.

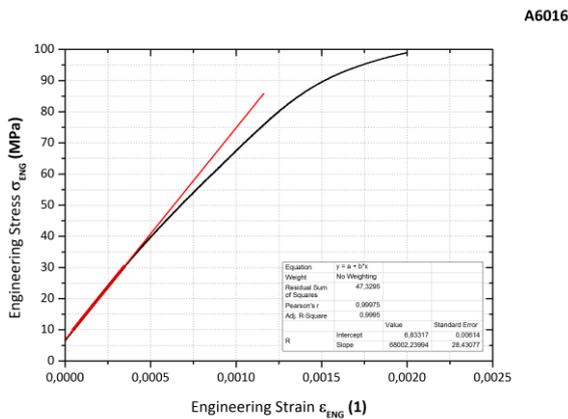


Figure 3. A6016 (static tensile test) – slope of  $\sigma_{ENG} - \epsilon_{ENG}$  CURVE

#### 3.2 Evaluation of the 3-point bend test (3PBT)

Young's modulus from the 3-point bend test can be calculated acc. to equation (1). Distance between supports is given by standard (in this case L = 45 mm).

$$E = \frac{L^3}{4at^3} \cdot k \quad (1)$$

- where:
- L - distance between supports (mm),
  - a - width of specimen (mm),
  - t - thickness of specimen (mm),
  - k - slope of F-y dependence (N·mm<sup>-1</sup>).

Beside slope k, all quantities are constants arising right from geometry. In fig.4 is shown the linear fitting to determine slope k for the 1<sup>st</sup> specimen (as a result k = 207,82757 N·mm<sup>-1</sup>). The used y-range was 5 N to 25 N and the same range was subsequently used for all measured specimens.

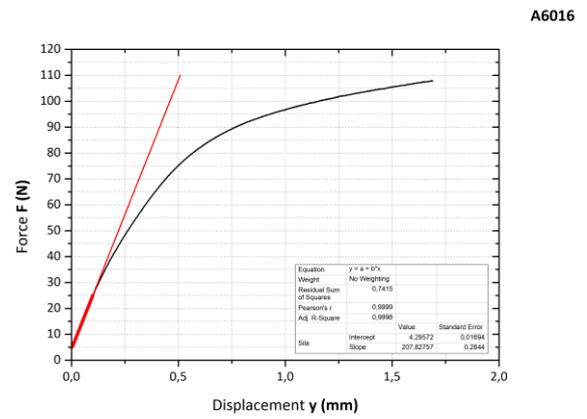


Figure 4. A6016 (3-point bend test) – slope of F - y curve

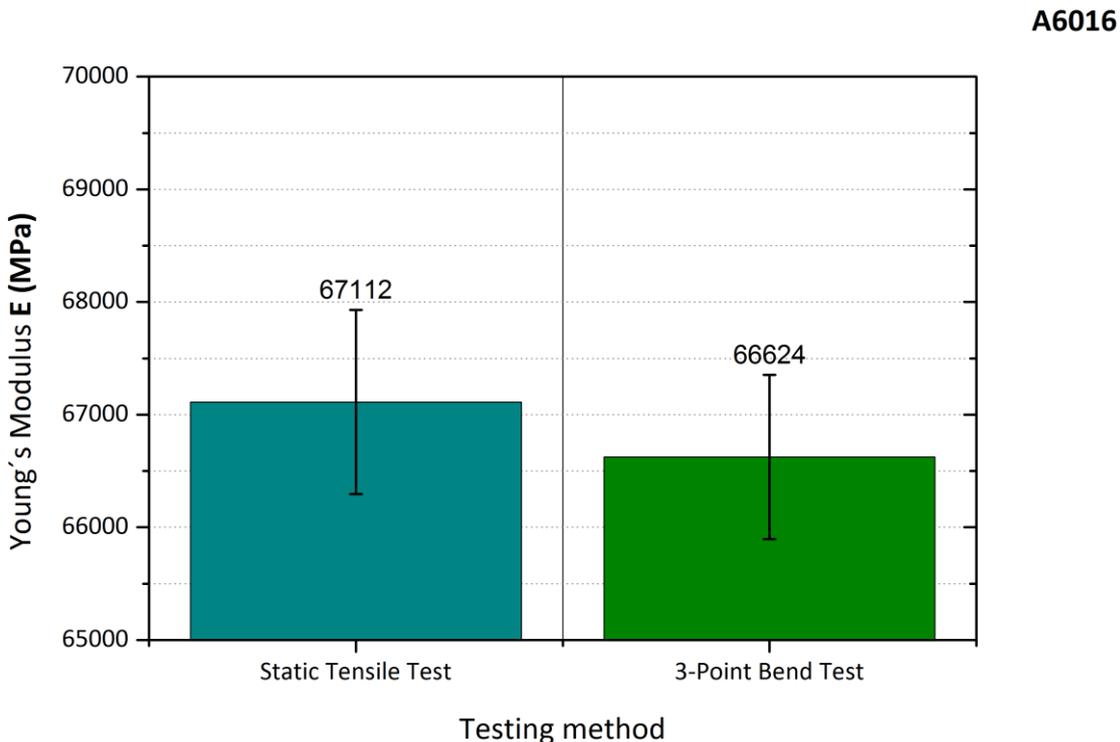


Figure 5. Statistical comparison (by mean and standard deviation) of Young's modulus vs. testing method for A6016

#### 4 DEEP-DRAWING MATERIAL DC06

The same testing methods under the same conditions were subsequently used also for the deep-drawing material DC06. That is why in this chapter are mentioned just basic differences in the evaluation methods. Again, basic statistical evaluation is graphically shown in fig. 8 (by mean and standard deviation).

##### 4.1 Evaluation of the static tensile test (STT)

Under the same testing conditions were measured engineering stress-strain curves for 10 specimens. Within the given y-range (in this case from 20 MPa to 60 MPa) was again used the linear fitting. In fig. 6 is shown such linear fitting for the 1<sup>st</sup> specimen where slope is right Young's modulus (here  $E = 191842$  MPa).

##### 4.2 Evaluation of the 3-point bending test (3PBT)

In the case of 3-point bend test was again used equation (1) and distance between supports was  $L = 42,5$  mm. Given y-range was from 4,5 N to 7 N. Fig. 7 illustrates that range and applied linear fitting for the 1<sup>st</sup> specimen (slope  $k = 70,76441$  N·mm<sup>-1</sup>). Geometry of specimens was the same as in the previous case.

#### 5 OVERVIEW OF FINAL RESULTS

In tab.3 are summarized all important results.

No. of specimen	A6016		DC06	
	STT	3PBT	STT	3PBTs
1	68002	66808	191842	188029
2	67824	65267	196757	187452
3	66339	67191	193897	191396
4	66039	66675	194215	185665
5	66656	67082	191551	186918
6	67042	67180	196634	187802
7	68684	65631	196361	192143
8	66805	66016	195279	188652
9	66972	67334	194987	189226
10	66758	67061	190642	188066

Table 3. Final results both for A6016 and DC06

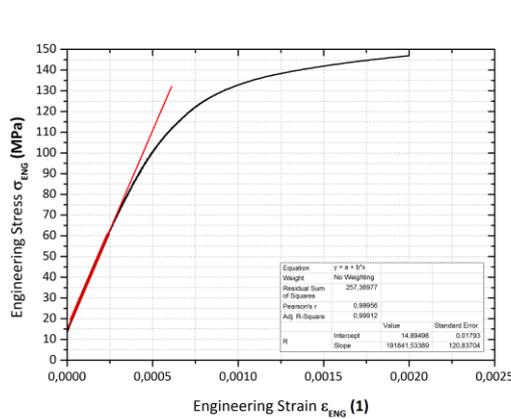


Figure 6. DC06 (static tensile test) – slope of  $\sigma_{ENG}$ -  $\epsilon_{ENG}$  curve

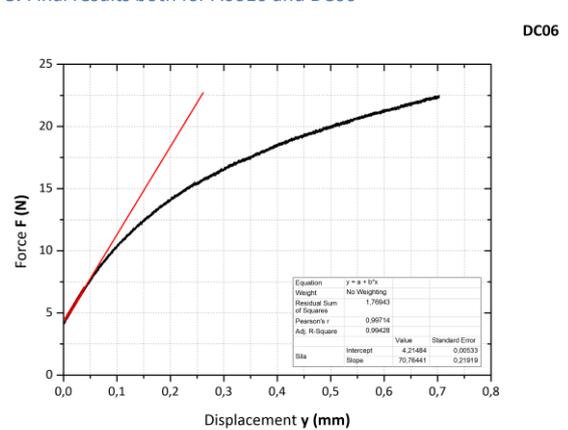


Figure 7. DC06 (3-point bend test) – slope of F - y curve

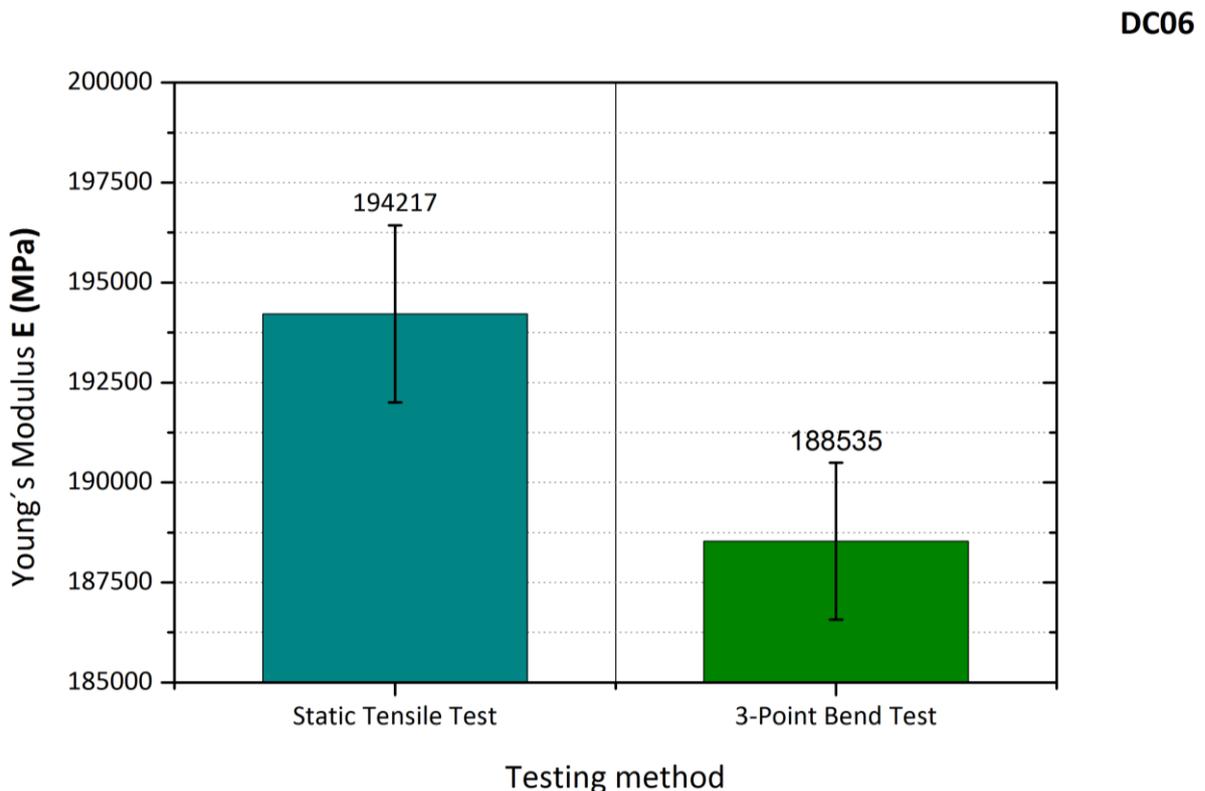


Figure 8. Statistical comparison (by mean and standard deviation) of Young's modulus vs. testing method for DC06

## 6 HYPOTHESIS TESTING

The major aim of the statistical evaluation was to determine if differences of means are/aren't significantly different.

**A6016:** the following fig. 9 graphically illustrates the individual results for all 10 used specimens of A6016.

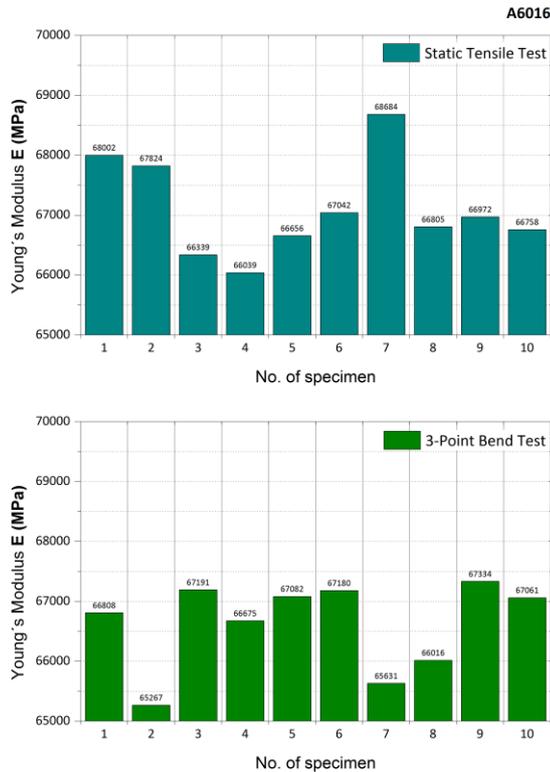


Figure 9. Overview of the individual results – A6016

Firstly there was necessary to carry out F-test (Fisher test) to determine the equality of two standard deviations. After that there was performed Students' t-test to determine if the difference of means is significantly different at the 0.05 level. All important input data are summarized in tab.4.

A6016	Static Tensile Test		3-Point Bend Test	
	X	S	X	S
E (MPa)	67112	816	66624	727

Table 4. Statistical evaluation of the aluminium alloy A6016

**F-test** (equality of two standard deviations)

- testing criterion:  $F = 1,25247$
- probability > F:  $p(F) = 0,74283$

From the result is evident that at the 0.05 level, the two measured variances are not significantly different and can be taken as homogenous variances.

**Student's t-test** (difference of means)

- testing criterion:  $t = 1,4117$
- probability > t:  $p(t) = 0,17501$

From the result of probability (0,17501) is evident that at the 0.05 level, the difference of two measured means **are not significantly different**. It can be stated that measurement of the Young's modulus **was not influenced** by the selection of testing method (static tensile test and 3-point bend test).

Difference of means for DC06 (see table 5) was quite large so it was truly very interesting to carry out Student's t-test.

**DC06:** the following fig. 10 again graphically illustrates the individual results for all 10 used specimens of DC06.

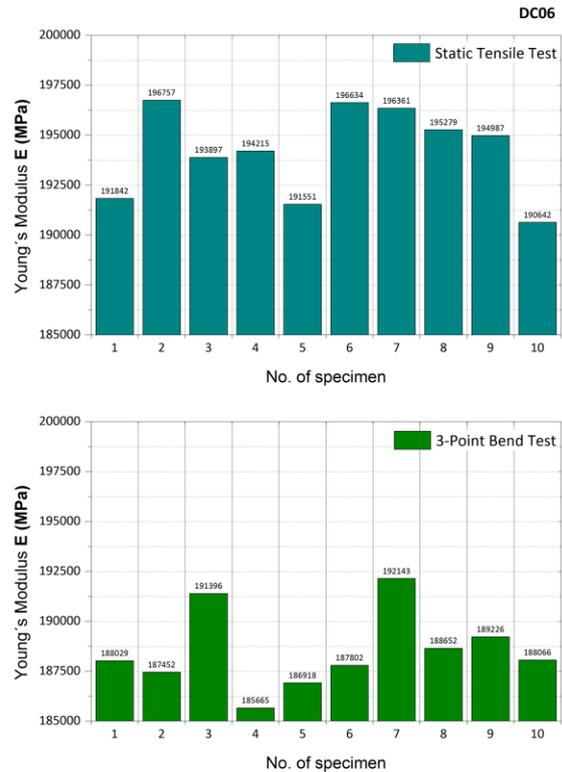


Figure 10. Overview of the individual results – DC06

Also in this case there was firstly carried out F-test (Fisher test) to determine the equality of two standard deviations. After that there was performed Students' t-test to determine if the difference of means is significantly different at the 0.05 level. All important input data are summarized in tab.5.

DC06	Static Tensile Test		3-Point Bend Test	
	X	S	X	S
E (MPa)	194217	2217	188535	1964

Table 5. Statistical evaluation of the deep-drawing material DC06

**F-test** (equality of two standard deviations)

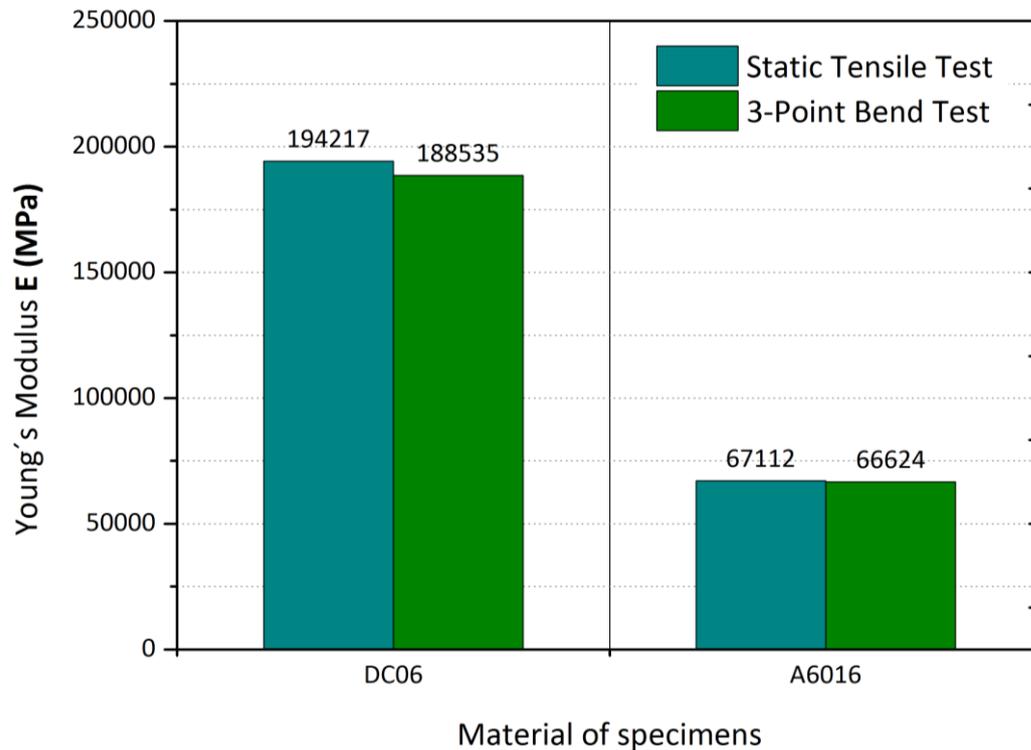
- testing criterion:  $F = 1,27412$
- probability > F:  $p(F) = 0,72407$

From the result is evident that at the 0.05 level, the two measured variances are not significantly different and can be taken as homogenous variances.

**Student's t-test** (difference of means)

- testing criterion:  $t = 6,06519$
- probability > t:  $p(t) = 0,00009864$

From the result of probability (0,00009864) is evident that at the 0.05 level, the difference of two measured means **are significantly different**. It can be stated that measurement of the Young's modulus **was influenced** by the selection of testing method (static tensile test and 3-point bend test).



**Figure 11.** Comparison of Young's modulus in light of used materials and testing methods

## 7 CONCLUSIONS

This paper deals with the different testing methods to determine modulus of elasticity (Young's modulus)  $E$  (MPa). As a first testing method there was used the static tensile test equipped with extensometer of high accuracy and it served as a reference testing method. As a second testing method there was used 3-point bend test which is sometimes used during production as fast (and also cheap of equipment) method to verify material properties. The main aim of this paper was to evaluate if differences of means are/aren't significantly different based upon the chosen testing method. As a testing materials there were used two different materials - aluminium alloy A6016 and common deep-drawing material DC06.

As was already written above, the most interesting result from the experimental part arose from the Student's  $t$ -test for differences of means. For aluminium alloy A6016 was proved that there is not influence of testing method on the final value of  $E$ . However, for deep-drawing material DC06 was proved that there is such influence (it was significantly different). According to monitoring of testing, such reality is probably resulting from quite small thickness of DC06 (0,710 mm). In standard is thickness taken into account for computation distance between supports. Nevertheless, it seems that especially in this case of thin deep-drawing material, results were influenced from the initial bending of sample and are not reliable. Thus as a recommendation for future research there is to carry out more experiments with such type of material and different thickness to prove observed results and such conclusion.

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