

# STUDY OF WEIGHTED MULTILATERATION IN VOLUMETRY OF CNC MACHINE TOOLS

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This paper describes utilization of a multilateration principle in the multi axis machine error modeling. Multilateration plays an important role in the identification of the exact position of the point in a machine tool workspace. The measured data were obtained using laser tracer in spherical coordinates. The multilateration attempts to improve the data accuracy using only a radial component. Two approaches are presented: the multilateration without weighing and the multilateration with weighing. That is based on the quality of the laserTRACER signal during the measurements. The weight of each value of the measured distance was given as a ratio of each signal quality to the best signal quality which was obtained during the measurement. Both of our results were compared to the data that were obtained by the original software TRAC-CAL.

## KEYWORDS

accuracy of measurement, multilateration, weighted multilateration, machine tool, LaserTRACER

## 1 INTRODUCTION

Nowadays, the demands on the accuracy and efficiency of machine tools are increasing. The main sources of uncertainty in machine tools (MT) are geometric errors, thermal errors and errors induced by cutting-force. We can classify two basic types of sources: quasi-static errors and dynamic errors. Quasi-static errors cause about 60-70% of the total error of the MT [Ramesh 2000]. This is the main inspiration for many research papers e.g. [Hrdina 2015] [Holub 2015] [Hrdina 2017]. When using the approach to geometric errors relative to work space, we talk about volumetric accuracy of the MT, which is given by volumetric deviation. The volumetric errors ( $ve$ ) are described as a difference between the pairs of a nominal point given by a numerical control and the real position of the point in work space of MT, the mean is given [Aguado 2014]

$$ve_{gn} = \frac{\sum_i^n \sqrt{(x_i - x_m)^2 + (y_i - y_m)^2 + (z_i - z_m)^2}}{n}, \quad (1)$$

where index  $t$  denotes theoretical values introduced by the machine setting and  $m$  is the index of the corresponding measured values. We consider  $ve$  as a combination of all geometric errors. If we want to be precise, we must assume an influence of heat sources [Mayr 2012] - thermal error ( $ve_t$ ), thus,  $ve = ve_g \pm ve_t$ . However, in this study the influence of heat sources was neglected and  $ve = ve_g$ . When the deviations are

characterized, their impact can be compensated. Some of new MT have already been equipped with a software calibration based on the volumetric verification. These compensations are based on various principles, for example using a laser measuring device [Bracat 2000] [Pedone 2014]. The presented paper describes the determination of a real position of the point in work space by using the technique of multilateration. Volumetric error was evaluated based on this value. One of the main goals is to show the influence of weighting on the final deviation.

## 2 THE MACHINE DESCRIPTION

The measured data were obtained using a laserTRACER (LT). This is a portable measurement system (see Fig.1.) that determines the position of the measured point in spherical coordinates. LT used in our study has following technical specifications: resolution of 1nm; uncertainty  $0.2 \mu\text{m} + 0.3\mu\text{m/m}$ ; measurement range of 15 m, arbitrarily by computational overlaying; temperature-invariant structure [AfM 2007-2017].



Figure 1. Etalon laserTRACER [AfM 2007-2017]

Our algorithm was prepared to be applied and verified on data from the vertical machining center MCV 754 QUICK. The work space (see Fig.2.) is defined by the Cartesian coordinate system  $x, y, z$  with working range of  $754 \times 500 \times 550$  mm. A table clamping surface has area of  $1000 \times 500$  mm [Kovosvit 2016].



Figure 2. Working space of vertical machining center MCV 754QUICK

### 3 PRINCIPLES OF MULTILATERATION

The measured data were obtained in the spherical coordinates. However, we need to eliminate the possible sources of inaccuracy in the measurement. In the case of the laser interferometer, there are many different disturbing factors including environmental errors, geometric errors or instrument errors. In order to enhance precision of the measurements, a necessity of monitoring environmental conditions such as air pressure, ambient temperature, and humidity (this alters the refractive index of measuring beam) arises [Teoh 2002].

To maximize the accuracy of the measuring instruments, the effects of noise caused by the inaccuracy of the interferometer or angle encoders needs to be reduced. For each point in the spherical coordinates there are two possible errors of angle components (azimuth, polar) and a radial component, see Fig. 3. The aim of multilateration is to reduce the measurement uncertainty, therefore only the radial component of each point is used. We have to use three or four LTs. The accuracy of location also depends on the number of LT's and their location [Takatsuji 2000]. In the next part of our study, we consider ideal location of LTs.

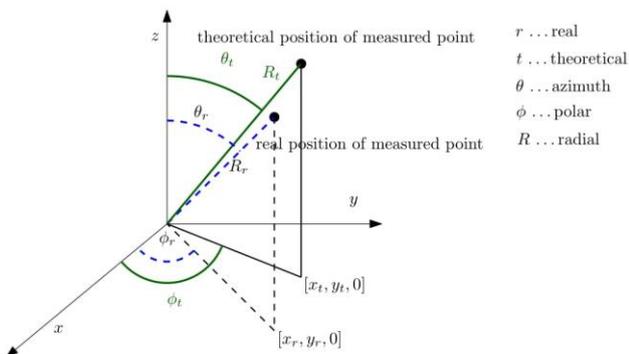


Figure 3. Measurement noise

When the point of the work space is measured with three LT's, this point is represented in three different LT coordinate systems CSLT<sub>k</sub> (k = 1; 2; 3). This representation of a measured point is transformed from CSLT<sub>k</sub> to the new coordinate system (NCS) (self-calibration) [Aguado 2014]. Three close but different points are obtained due to the noise of measurement. The accurate position of the point is given by the intersection of three spheres. Each of the spheres is defined by radial component of each LT<sub>k</sub>. Respectively, therefore points [x<sub>k</sub>, y<sub>k</sub>, z<sub>k</sub>] are the centers of the spheres with radii given by r<sub>k</sub>

$$r_k^2 = (x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2 \quad (1)$$

When the origin of NCS is identified with one of the CSLT<sub>k</sub> (for example CSLT<sub>1</sub>), the coordinates of the point are given by the system of equations of this form [Aguado 2014]

$$\begin{aligned} x &= (r_1^2 - r_2^2 - x_2^2) / (2x_2) \\ y &= (r_1^2 - r_3^2 + x_3^2 + y_3^2 - 2xx_3) / (2y_3) \\ z^2 &= r_1^2 - x^2 - y^2 \end{aligned} \quad (2)$$

The signum of z coordinate can be given by fourth LT [Aguado 2014].

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} 1 & 0 & 0 \\ x_2 & 1 & 0 \\ -x_3 & y_3 & 1 \\ x_2 y_3 & y_3 & 1 \\ -x_4 + x_3 y_4 & -y_4 & 1 \\ x_2 z_4 & x_2 y_3 z_4 & y_3 z_4 & z_4 \end{bmatrix} \begin{bmatrix} r_2^2 - r_1^2 - x_2^2 \\ r_3^2 - r_1^2 - x_3^2 - y_3^2 \\ r_4^2 - r_1^2 - x_4^2 - y_4^2 - z_4^2 \end{bmatrix} \quad (3)$$

### 4 PROBLEM FORMULATION

In our case, we used only one LT in four different positions for each measurement. The data are obtained for each point [x<sub>i</sub>, y<sub>i</sub>, z<sub>i</sub>] of the work space e.g. see Tab.1.

x coordinate [mm]	y coordinate [mm]	z coordinate [mm]	Length [mm]	quality [%]
225	0	0	531.500430	80
300	0	0	585.298301	83
375	0	0	643.482860	85

Table 1. Input data of position 1

We used the principles of multilateration for the input adjustments. First, we determined the exact position of each LT. In this case, input parameters are coordinates of all the points introduced by numerical model and distances measured between LT on each position and the measured point. The object of minimization is the LT's position. This problem was solved [Navratilova 2016] by least square method

$$\min_x \|F(x)\|_2^2 = \min_x \sum_i F_i^2(x), \quad (4)$$

where

$$F_i = \sum_{k=1}^4 (\sqrt{(x_i - LT_x^k)^2 + (y_i - LT_y^k)^2 + (z_i - LT_z^k)^2} - \dots - (LT_D^k - Pk_i))^2, \quad (5)$$

where index k indicates the position of LT, point [LT<sub>x</sub>, LT<sub>y</sub>, LT<sub>z</sub>] is the position of LT, index D denotes dead distance.

We consider vector qk (k = 1, ..., 4) of value of quality of TRACER signal of LT<sup>k</sup>. For each measured distance of each LT<sup>k</sup> was introduced weight

$$w_i = \frac{qk_i}{\max_i \bigcup_k qk_i} \quad (6)$$

To apply this weight to our problem, the function F<sub>i</sub> was changed in following form

$$F_i = \sum_{k=1}^4 (N^k)^2 w_i, \quad (7)$$

where

$$N^k = \sqrt{(x_i - LT_x^k)^2 + (y_i - LT_y^k)^2 + (z_i - LT_z^k)^2} - (LT_D^k - Pk_i), \quad (8)$$

When the position of each LT was known, we applied the same process to determine the exact position of point in work space.

## 5 RESULTS AND DISCUSSION

Previous formulation of the problem was simulated in MATLAB. The trust-region-reflective least squares algorithm was used to provide better stability.

First, we used our algorithm for determination of each LT position. Following table (Tab.2.) compares our results with results obtained by software TRAC-CAL.

position	x coordinate [mm]	y coordinate [mm]	z coordinate [mm] (with offset)
1_TR	-113.51	130.50	-479.64
1_N	-113.5257	130.5018	-479.6422
1_W	-113.5278	130.5024	-479.6399
2_TR	-114.57	305.53	-479.56
2_N	-114.5718	303.5299	-479.5617
2_W	-114.5680	303.5296	-479.5581
3_TR	790.27	126.77	-350.21
3_N	790.2745	126.7693	-350.2127
3_W	790.2667	126.7720	-350.2092
4_TR	802.65	308.95	-350.19
4_N	802.6507	308.9533	-350.1950
4_W	802.6453	308.9544	-350.1917

**Table 2.** position of LT index TR indicates TRAC-CAL™ date, N our result without using weight, W our results with using weight, offset = -122 [mm]

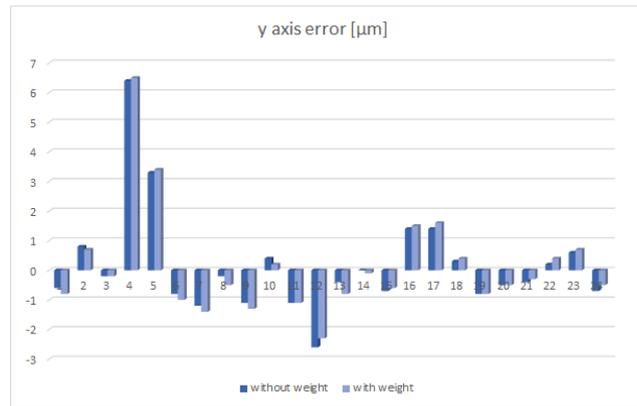
In the second step, the input parameters are: position of each LT and measured distances. Positions of all points in the workspace are minimized in this step. Following figures illustrate the influence of selected weighting on the final position error.



**Figure 4.** x axis errors [μm] for point 1-24 in work space

x	y	z	error	error_w
225	0	0	0.0024	0.0021
300	0	0	0.0012	0.0010
375	0	0	0.0010	0.0008

**Table 3.** x-axis compression between result of error without weight (error) and with weight (error\_w) [mm]

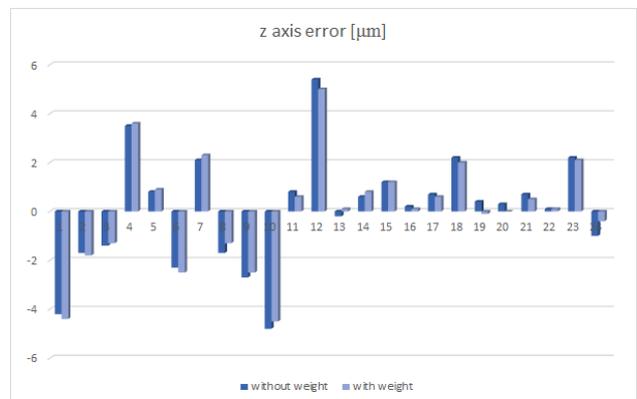


**Figure 5.** y axis errors [μm] for point 1-24 in work space

x	y	z	error	error_w
225	0	0	-0.0006	-0.0008
300	0	0	0.0008	0.0007
375	0	0	-0.0002	-0.0002

**Table 4.** y-axis compression between result of error without weight (error) and with weight (error\_w) [mm]

As seen, the difference of those two approaches may be up to 0.5μm. For better illustration, each axis error is shown separately. Figure 4 shows x component of volumetric error, Fig.5 shows y component and on the Fig.6 z component. Each table (Tab.3, Tab.4, Tab.5) shows the exact value of error components.



**Figure 6.** z axis errors [μm] for point 1-24 in work space

x	y	z	error	error_w
225	0	0	-0.0042	-0.0044
300	0	0	-0.0017	-0.0018
375	0	0	-0.0014	-0.0013

**Table 5.** z-axis compression between result of error without weight (error) and with weight (error\_w) [mm]

## 6 CONCLUSIONS

This paper is devoted to the determination of the points positions in the work space by using the multilateration technique. Two approaches are presented – without and with weighing. In the second case, we considered the influence of the quality change of the laserTRACER signal during the

measurements. In this case, weights were given for each measured distance.

The presented algorithm works in two steps: determination of the exact position of LT and determination of the exact position of the points in the work space. Regarding the first step, we compared both approaches with the data that were obtained by the original software TRAC-CAL. The results presented in Tab.2 (a comparison between TRAC-CALTM data and data enumerated by our algorithm) show that the difference between our considerations and the corresponding position obtained by TRAC-CAL are negligible. In terms of those information, we proved correctness of our method. In the second step, we compared our results only. In our future work, we will consider including another influence of the measurement in model precision.

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