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FORCE ESTIMATION MODEL AND PARAMETER IDENTIFICATION IN END MILLING PROCESS WITH CORNER EDGE DAMAGE

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Abstract
Identification of the appropriate model parameters in the cutting process is essential to achieve highly accurate cutting simulations that are effective for the process optimization. During end milling, tool damage deteriorates not only the side cutting edge geometry but also the corner cutting edge as the cutting distance increases. The conventional model does not consider the effect of local corner edge damage on the cutting force. Therefore, we proposed a model and its parameter identification method considering the corner edge damage. The tool life test verified that the corner damage modeling significantly improves the force estimation accuracy.

Keywords:
milling, force model, corner edge damage, tool wear, parameter identification

1 INTRODUCTION

To achieve high performance cutting, it is necessary to select appropriate machining conditions. Simulations that take into account machining characteristics are effective for process optimization [Altintas 2014]. Proper modeling and accurate parameter identification are essential to achieve highly accurate simulation. A variety of models have been proposed to estimate the cutting forces in machining with square end mills [Usui 1978] [Altintas 2012] [XF. Zhao 2021] [E. Budak 1996]. It is known that the cutting force can be modeled as the sum of the chip generation force and edge force components. As the cutting distance increases, the cumulative tool damage deteriorates the tool cutting edge geometry. The influence on the cutting force is quantitatively represented by the change in the cutting force coefficients as reported in [Liu Y 2022] [Oraby SE 1991]. Mostly, the previous studies focused on the damage generated on the side cutting edge of the end mills. However, the damage generated on the end mill corner becomes greater than that on the side cutting edge. Ignoring this specific corner damage may result in a decrease in the accuracy of cutting force estimation. To clarify the influence of the corner modeling, we proposed a new cutting force model that considers the influence of the corner edge and the identification method of the model parameters. A series of experiments verified that tool defects at the corner edge have a significant influence on the cutting force estimation.

2 MODELING OF THE MILLING PROCESS CONSIDERING THE EFFECTS OF TOOL WEAR

The milling process model with a square end mill is shown in Fig. 1. For the cutting force estimation, the mechanistic model [Altintas 2012] is employed. The cutting force is divided into the cutting resistance component, which contributes to chip generation, and the edge force component, which contributes to ploughing the workpiece at the cutting edge. In machining with square end mills, as the cutting distance increases, the geometric deterioration occurs on not only the side cutting edge but also the bottom edge corners. Previous studies only focused on the effects of tool wear occurring on the side flutes of the end mill [Fujii 2023]. On the other hand, the influence of the corner edge defect has not received much attention. In the present study, we focus on the tool defect at the corner edge and propose a milling force model that takes into account the corner edge deterioration.

In general, in the end mill cutting model, the end mill is divided along the axial direction and the cutting force generated on each small cutting edge is calculated. The total cutting force is then estimated by accumulating all components at the cutting edges involved in cutting. In this study, the processes by corner edge were considered independent of the side cutting edges and its cutting force was integrated with that of the side cutting edge. Assuming that the micro cutting width is Δa, the uncut chip thickness is h, the specific cutting force and edge force coefficients of the side edges are $K_{\mathrm{ct}}, K_{\mathrm{cr}}, K_{\mathrm{ca}}$ and $K_{\mathrm{et}}, K_{\mathrm{er}}, K_{\mathrm{ea}}$, respectively. The specific cutting resistance and edge force components of the corner edge are $K_{\mathrm{bt}}, K_{\mathrm{br}}, K_{\mathrm{ba}}$ and $F_{\mathrm{bt}}, F_{\mathrm{br}}, F_{\mathrm{ba}}$. The cutting force is expressed by equation (1).

$$\boldsymbol{F}_c = \sum_j^{N_j} \left(\sum_l^{N_a} \Delta a \boldsymbol{T}_{j,l} \begin{bmatrix} K_{ct} h_{j,l} + K_{et} \\ K_{cr} h_{j,l} + K_{er} \\ K_{ca} h_{j,l} + K_{ea} \end{bmatrix} + \boldsymbol{T}_{j,1} \begin{bmatrix} K_{bt} h_{j,1} + F_{bt} \\ K_{br} h_{j,1} + F_{br} \\ K_{ba} h_{j,1} + F_{ba} \end{bmatrix} \right) \quad (1)$$

The nominal feed per tooth is c and the tool eccentricity components are Δx,Δy. Considering the eccentricity, the cutting thickness $h_{j,l}$ at the micro cutting edge at the l-th axial position on the j-th cutting edge is approximated by equation (2).

$$h_{j,l} \approx c \sin \phi_{j,l} + \Delta x\left(\sin \theta_{j,l} - \sin \theta_{j-1,l}\right) + \Delta y\left(\cos \theta_{j,l} - \cos \theta_{j-1,l}\right) \quad (2)$$

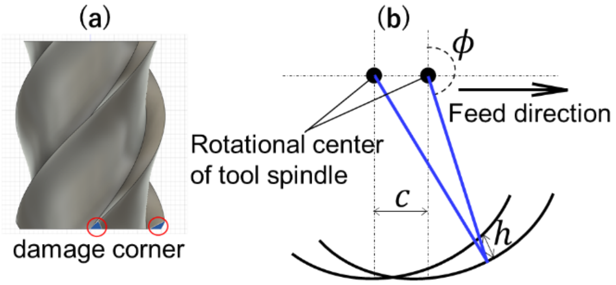


Fig. 1: Milling process model; (a) damage corner (b) uncut chip thickness.

3 IDENTIFICATION OF MODEL PARAMETERS

We developed two different parameter identification methods by utilizing time-averaged or dynamic forces measured during end milling. "Time-averaging method" is featured by its robust nature against dynamic measurement noise [E. Budak 1996]. On the contrary, "dynamic method" applies model-based inverse analysis to time-varying force data. The linear equations for time-averaging method are shown in Equations (3) and (4).

$$[\overline{F_1} \quad \overline{F_2}]^T = [\boldsymbol{A}]\boldsymbol{X_1} \quad (3)$$

$$[\overline{F_1}]^T = [\boldsymbol{A'}]\boldsymbol{X_2} \quad (4)$$

$$\boldsymbol{X_1} = [K_{ct}\ K_{cr}\ K_{ca}\ K_{et}\ K_{er}\ K_{ea}\ K_{bt}\ K_{br}\ K_{ba}\ F_{bt}\ F_{br}\ F_{ba}] \quad (5)$$

$$\boldsymbol{X_2} = [K_{ct}\ K_{cr}\ K_{ca}\ K_{et}\ K_{er}\ K_{ea}] \quad (6)$$

$\bar{F}$ is the time-averaged cutting force and A is coefficient matrix that depends on the machining conditions. The averaging method identifies the parameter vector $\boldsymbol{X_1}$ by the proposed method or $\boldsymbol{X_2}$ by the conventional method. It is necessary to provide appropriate reference force data to satisfy the full rank to properly weight the influence of the chip generation force and edge force. Therefore, $\overline{F_1}$ needs to contain data under several conditions with different feed rates. It should be noted that the reference data $\overline{F_2}$ at the different axial depths of cut are simultaneously needed to identify the corner edge parameters in the proposed model.

Next, we consider the dynamic method, which is partially explained in [Takahei 2023] [Miwa 2021]. The linear equations for the dynamic method are shown in Equations (7), (8). $\boldsymbol{F_c}$ is the dynamic cutting force and $\boldsymbol{X_1}, \boldsymbol{X_2}$ are the parameter vectors. The force ratio $k_{r1}(= K_{cr}/K_{ct})$, $k_{a1}(= K_{ca}/K_{ct})$, $k_{r2}(= K_{br}/K_{bt})$ and $k_{a2}(= K_{ba}/K_{bt})$ are defined.

$$\boldsymbol{F_c} = [\boldsymbol{B}]\boldsymbol{X_3} \quad (7)$$

$$\boldsymbol{F_c} = [\boldsymbol{C}]\boldsymbol{X_4} \quad (8)$$

$$\boldsymbol{X_3} = [K_{ct}\ K_{ct}\Delta x\ K_{ct}\Delta y\ K_{et}\ K_{er}\ K_{ea}\ K_{ct}\ K_{bt}\Delta x\ K_{bt}\Delta y\ F_{bt}\ F_{br}\ F_{ba} \quad (9)$$

$$\boldsymbol{X_4} = [k_{r1}\ k_{a1}\ k_{r2}\ k_{a2}] \quad (10)$$

In the both method, the model parameters are identified by least squares method. The milling process model can't be treated as a perfectly linear system due to its nonlinearity. Hence, in the dynamic method, identification calculations are iteratively performed by updating initial parameters. The dynamic method essentially does not require cutting force information under the different conditions of feed rate or axial cutting depth to satisfy the ranks in the parameter identification because it includes dynamic information at different moments of the cutting edge contact area. On the other hand, identification accuracy may depend on the dynamic characteristics of the force measurement means.

4 VALIDATION OF THE PROPOSED METHOD BY MILLING EXPERIMENTS

We validated the proposed method by varying the tool geometry through tool life tests. In the verification experiment, the cutting forces were measured up to a cutting distance of 120 m by repeatedly machining the carbon steel workpiece (S45C) using a two-flute coated carbide tool with a diameter of 10 mm. Firstly, we compared the transition of the error norm in time-averaging method to verify the accuracy of the model. Secondly, we performed time-domain simulations of cutting forces from the parameters obtained with each identification method to compare the identification accuracy of the time-averaging and dynamic methods. In the comparison, the identification accuracy is compared referring the different reference data to investigate the influence of the experimental conditions. The main experimental conditions are listed in Tab. 1. These experiments are aimed to clarify the questions listed in Tab. 2.

Tab. 1: Cutting conditions.

Number of flutes	2
Tool diameter	10 mm
Axial depth of cut	4 and 7.5 mm
Radial depth of cut	1 mm
Spindle speed	4800 rpm
Feed rate	200, 300, 400, 500, 600 mm/min

Tab. 2: Analytical approaches to the questions.

Questions	Analytical approaches
Influence of modeling error?	Comparison of error norms in conventional and proposed models.
Discrete vs Average?	Comparison of error norms for time-domain simulations of cutting force.
Influence of experimental conditions?	Comparison of error norm for axial depth of cut conditions.

4.1 Influence of modeling error

Fig. 2 shows the variations of the error norms in the time-averaging method. The experimental results indicate that the instantaneous damage occurred at the bottom corner edge when the cutting distance exceeded 50 m. The error norm of the proposed model that considers the effect of the corner edge is $||\boldsymbol{e_1}||$, and that of the conventional model that does not consider the effect of the corner edge is $||\boldsymbol{e_2}||$. The error norms are calculated referring the results measured at two different axial depth of cut. Fig. 2 shows that the proposed method has a smaller increase in the error norm as compared with the conventional method. In other words, the accuracy of cutting force estimation can be improved by considering the corner edge deterioration. Fig. 3 shows that the specific cutting resistance and edge force of the bottom edge increase rapidly after a cutting distance of 60m. This suggests that the effect of wear and tear on the bottom edge corner was pronounced. The axial specific cutting resistance K_{ba} and edge force F_{ba} of the bottom edge show a decreasing trend. This is due to an increase in the surface crushing force in the bottom edge wear, which is expected to correlate with the degradation process of the finished surface.

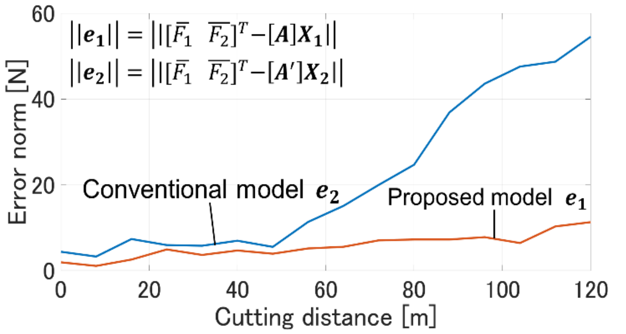


Fig. 2: Comparison of the transition of the error norm in the time-averaging method.

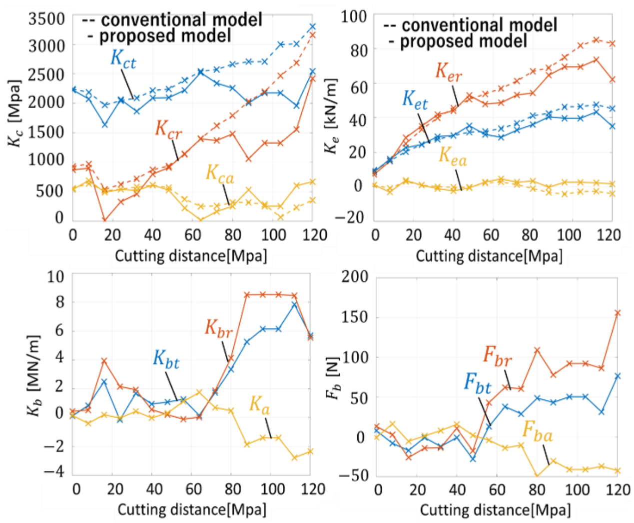


Fig. 3: Transition of Identification Parameters Obtained by the time-averaging Method.

4.2 Comparison of identification accuracy between dynamic and time-averaging methods and influence of reference force data

The identification accuracy of the dynamic and time-averaging methods in the proposed model is compared. In addition, the influence of reference data on the identification accuracy is investigated. The error norm of time-domain simulations with respect to the measured dynamic data were compared. The result of the comparison is shown in Fig. 4. To investigate relative difference, the error norms are normalized based on the results of the error norm at a cutting distance of 0 m with two axial depths of cut $||{}^{dm}_{0}e_d||$. The normalized error norm $||\boldsymbol{e}_{n3}||$ is formulated in Equation (12) from the error norm at each cutting distance $||\boldsymbol{e}_3||$ in Equation (11).

$$\boldsymbol{e_3} = \boldsymbol{F_{c1}} - [\boldsymbol{B_1}]\boldsymbol{X_3} + \boldsymbol{F_{c2}} - [\boldsymbol{B_2}]\boldsymbol{X_3} \tag{11}$$

$$\text{Normalized error norm: } ||\boldsymbol{e_{n3}}|| = \frac{||e_3||}{||{}^{dm}_{0}e_d||},$$
$$||{}^{dm}_{0}e_d|| = \boldsymbol{F}_{c1} - [\boldsymbol{B_1}{}^{dm}_{0}\boldsymbol{X}_d + \boldsymbol{F}_{c2} - [\boldsymbol{B_2}]^{dm}{}^{dm}_{0}\boldsymbol{X}_d \tag{12}$$

The notation and corresponding formulas for the error norms are summarized in Tab. 3. In Fig. 4, we labelled the Dynamic method 'dm' and Time-averaging method 'am'. When the reference data consists only of axial depth of cut of 4 mm or 7.5 mm, it is denoted as s4 or s7.5, respectively. When the data consisted of both 4 mm and 7.5 mm cuts, they were denoted as d. Additionally, the normalized cases were denoted as ns4 or nd. For example, $||{}^{am}_{0}e_{ns7.5}||$ denotes the normalized error norm calculated using Time-averaging method and referencing only the 7.5 mm axial depth of cut data.

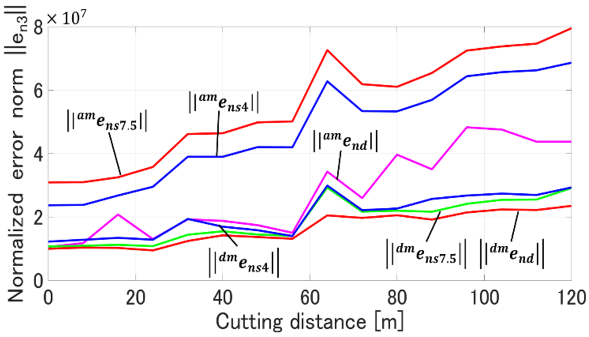


Fig. 4: Comparison of normalized error norms.

Calculation results indicate that the dynamic method has a smaller error norm as compared to the time-averaging method. This is mostly due to the consideration of tool eccentricity. The dynamic method shows no significant change in the error norm regardless of the reference data. On the other hand, in the case of the time-averaging method, the influence of reference data is significant. This fact suggests that the dynamic method only requires the simple reference data without variation in the axial depth of cut. In other words, the dynamic method is robust against the cutting conditions.

A comparison was made between cutting forces estimated using parameters identified by the dynamic method at cutting distances of 0 m and 120 m. The experimental results are compared to those estimated by the conventional model ignoring with the corner edge damage as shown in Fig. 5 and the proposed model in Fig. 6. A single reference data set was used for the parameter identification in each simulation in Fig. 5 and Fig. 6. Fig. 7 demonstrates a comparison between the estimated and experimental cutting forces obtained from two reference data set with different axial depth of cut using the proposed model. The experimental results with the axial depth of cut of 7.5 mm and cutting feed of 300 mm/min are demonstrated in Fig. 5, 6 and 7. Red line shows the estimated cutting force and blue line shows the experimental cutting force. Here, we compared the cutting forces in the z direction which are most affected by tool corner damage. As the cutting length increases, the cutting force in the z-direction becomes negative. This phenomenon is likely attributable to the increased downward thrust on the workpiece caused by damage to the corner cutting edge, as also indicated in Fig. 3. The figures indicate better fit in the proposed model which verifies higher estimation accuracy than the conventional model. Fig. 5 and Fig. 6 also show that the identification accuracy does not change depending on the axial depth of cut conditions. In other words, it is not necessary to include different axial depth of cut conditions in the reference data when using the dynamic method. Tab. 4 summarizes the identification accuracy for each identification method for each axial depth of cut conditions. In Tab. 4, o means good estimation accuracy, × for poor, and △ for intermediate. But it should be noted that the identification accuracy may depend on the force measurement accuracy.

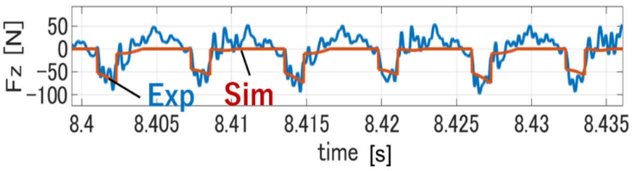


Fig. 6: Simulation using the proposed model referring 1 condition at the cutting distance of (up) 0 m and (down) 105 m.

Tab. 3: Definition of the normalized error norm

Identification method	Axial depth of reference data	Calculation of the error vector	Normalized error norm
Time-averaging method (am)	7.5 mm	$\|\|^{am}\boldsymbol{e}_{s7.5}\|\| = \boldsymbol{F}_{c1} - [\boldsymbol{B}_1]^{am}\boldsymbol{X}_{s7.5} + \boldsymbol{F}_{c2} - [\boldsymbol{B}_2]^{am}\boldsymbol{X}_{s7.5}$	$\|\|^{am}\boldsymbol{e}_{ns7.5}\|\| = \|\|^{am}\boldsymbol{e}_{s7.5}\|\| / \|\|^{dm}_{0}\boldsymbol{e}_d\|\|$
	4 mm	$\|\|^{am}\boldsymbol{e}_{s4}\|\| = \boldsymbol{F}_{c1} - [\boldsymbol{B}_1]^{am}\boldsymbol{X}_{s4} + \boldsymbol{F}_{c2} - [\boldsymbol{B}_2]^{am}\boldsymbol{X}_{s4}$	$\|\|^{am}\boldsymbol{e}_{ns7.5}\|\| = \|\|^{am}\boldsymbol{e}_{s4}\|\| / \|\|^{dm}_{0}\boldsymbol{e}_d\|\|$
	4, 7.5 mm	$\|\|^{am}\boldsymbol{e}_{d}\|\| = \boldsymbol{F}_{c1} - [\boldsymbol{B}_1]^{am}\boldsymbol{X}_{d} + \boldsymbol{F}_{c2} - [\boldsymbol{B}_2]^{am}\boldsymbol{X}_{d}$	$\|\|^{am}\boldsymbol{e}_{nd}\|\| = \|\|^{am}\boldsymbol{e}_{d}\|\| / \|\|^{dm}_{0}\boldsymbol{e}_d\|\|$
Dynamic method (dm)	7.5 mm	$\|\|^{dm}\boldsymbol{e}_{s7.5}\|\| = \boldsymbol{F}_{c1} - [\boldsymbol{B}_1]^{dm}\boldsymbol{X}_{s7.5} + \boldsymbol{F}_{c2} - [\boldsymbol{B}_2]^{dm}\boldsymbol{X}_{s7.5}$	$\|\|^{dm}\boldsymbol{e}_{ns7.5}\|\| = \|\|^{am}\boldsymbol{e}_{s7.5}\|\| / \|\|^{dm}_{0}\boldsymbol{e}_d\|\|$
	4 mm	$\|\|^{dm}\boldsymbol{e}_{s4}\|\| = \boldsymbol{F}_{c1} - [\boldsymbol{B}_1]^{dm}\boldsymbol{X}_{s4} + \boldsymbol{F}_{c2} - [\boldsymbol{B}_2]^{dm}\boldsymbol{X}_{s4}$	$\|\|^{dm}\boldsymbol{e}_{ns4}\|\| = \|\|^{am}\boldsymbol{e}_{s4}\|\| / \|\|^{dm}_{0}\boldsymbol{e}_d\|\|$
	4, 7.5 mm	$\|\|^{dm}\boldsymbol{e}_{d}\|\| = \boldsymbol{F}_{c1} - [\boldsymbol{B}_1]^{dm}\boldsymbol{X}_{d} + \boldsymbol{F}_{c2} - [\boldsymbol{B}_2]^{dm}\boldsymbol{X}_{d}$	$\|\|^{dm}\boldsymbol{e}_{nd}\|\| = \|\|^{am}\boldsymbol{e}_{d}\|\| / \|\|^{dm}_{0}\boldsymbol{e}_d\|\|$

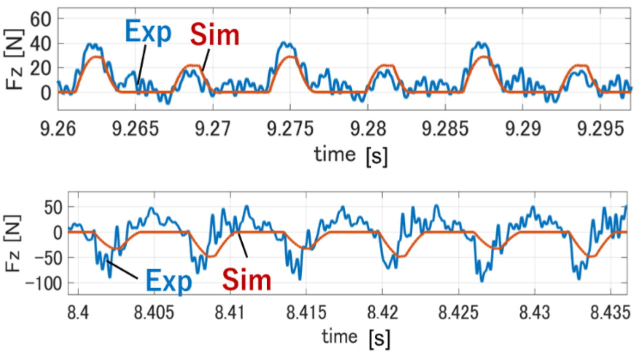


Fig. 5: Simulation using the conventional model at the cutting distance of (up) 0 m and (down) 105 m.

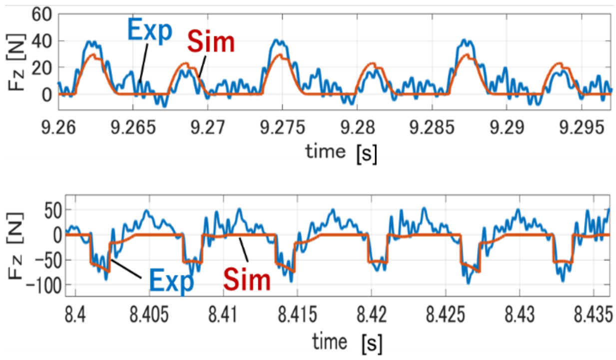
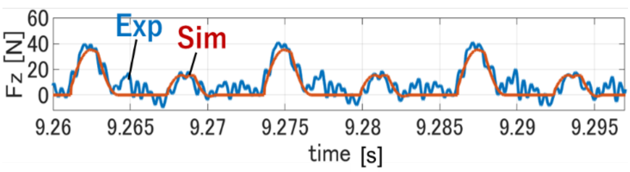


Fig. 7: Simulation using the proposed model referring 2 conditions at the cutting distance of (up) 0 m and (down) 105 m.

Tab. 4: Comparison of process simulation accuracy.

	Multi-reference data with different axial depth of cut	Reference data with a single axial depth of cut
Time-averaging method	△	×
Dynamic method	o	o



5 SUMMARY

In this study, we proposed a new cutting force model that takes into account the influence of corner edge deterioration of square end mills. Furthermore, we proposed a method for identifying the model parameters using the time-averaging method and the dynamic method. The results of tool life tests show that the proposed model significantly improves the accuracy of cutting force estimation. The dynamic method has a higher estimation accuracy of the identified parameters as compared to the time-averaging method. In addition, the dynamic method does not require multiple axial depth of cut data for its reference data. Hence, the proposed method could be practical for the milling process monitoring with the damaged end mills.

6 REFERENCES

Paper in a journal:

[Altintas 2014] Altintas Y, et al. Visual process systems for part machining operations, CIRP Annals, 2014, Vol.63, No.2, pp.585-605

[Budak 1996] E. Budak, et al. Prediction of Milling Force Coefficients From Orthogonal Cutting Data, Journal of Manufacturing Science and Engineering, May 1996, Vol.118, No.2, pp.216-224

[Fujii 2023] Fujii et al. Estimation of hardness and residual stress on the end milled surface by linear regression model, International Journal of Automation Technology, November 2023, Vol.17, No.6, pp.564-574

[Liu Y 2022] Liu Y, et al. Monitoring of in-process force coefficients and tool wear, CIRP Journal of Manufacturing Science and Technology, August 2022, Vol.38, pp.105-119

[Oraby S.E. 1991] Oraby S.E., et al. Development of models for tool wear force relationships in metal cutting, International Journal of Mechanical Science, 1991, Vol.33, pp.125-138

[Takahei 2023] K. Takahei, et al, Parameter identification for linear model of the milling process using spindle speed variation, Precision Engineering, January 2023, Vol.79, pp.16-33

[Usui 1978] Usui E, et al. Analytical Prediction of Three Dimensional Cutting Process—Part 1: Basic Cutting Model and Energy Approach, Journal of Engineering for Industry, May 1978, Vol.100, pp.222-228

Paper in proceedings:

[Miwa 2021] S. Miwa, et al. Analytical study on identification of milling simulation parameters by utilizing spindle speed variation method, LEM21, December 2021, pp.34-37, ISSN 24243086

Technical reports or thesis:

[XF. Zhao 2021] XF. Zhao, et al, Experiment and Modeling of Milling Force Based on Tool Edge Prediction. Experimental techniques, October 2021, Vol.46, pp.761-773

[Altintas 2012] Altintas Y. Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design, Cambridge University Press, 2012