DETERMINATION OF PARAMETER Ra FOR THE METHODS OF FINISHING MACHINING

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Theoretical determination of surface roughness Ra parameter for methods of machining tool with undefined geometry is complex. Paper presents an analysis of possible relationships to calculate the roughness parameters Ra finishing with rotating parts. Based on the selection of variables involved, wanted variables and factors to establish differential equation. The equation contains in addition of the factors also the constant, which must be established experimentally. For this case, it is appropriate to use the method of planned experiment, which would be extended to a new method for determining of various mathematical relations for Ra. The presented methodology can be applied to the determination of Ra with other methods of finishing machining, especially with the grinding.

Keywords:
Roughness, Experiment, Differential equation, Machining, Grinding

1. Introduction

A roughness of the surface after superfinishing and sharpening is one of the most important achieved parameters. A roughness has its own parameters also and the most widely used, and therefore the most important is the arithmetic mean deviation of the profile of the finished surface, i. e., the parameter Ra.

In the superfinishing the surface morphology and hence the parameter Ra are influenced especially by the time of the superfinishing t_r, the pressure of the instrument (a superfinishing stone) on the workpiece p_s and the speed ratio g (the ratio of the speed of an oscillating motion of the superfinishing tool to the peripheral speed of the workpiece). The granularity of the superfinishing stone we neglect, because if the process of the superfinishing is divided into roughing and finishing, the formula for the both cases would have to be derived. However, if we focus on the continuous superfinishing (the workpiece passes under roughing and finishing stones on one track) only one equation is set for the calculation of Ra with the respect to the granularity of roughing and finishing stones.

In the sharpening the parameter Ra is influenced mainly by the time of sharpening t_r and by the pressure of the tool on the workpiece p_s.

2. The source equations for the determination of the parameter Ra.

Actually the time decrease of the roughness is a negative first derivation of the parameter Ra according to the time of superfinishing $-\frac{dRa}{dt_r}$ 2008] [Skocovsky 1977]. This time decrease of the roughness parameter Ra may be proportional to the roughness itself, which is determined by the parameter Ra and so

$$-\frac{dRa}{dt_r} = a \cdot Ra \tag{1}$$

The solution of the differential equation [Liska , 2009] (1) is

$$\begin{aligned} \frac{dRa}{Ra} &= -a \cdot dt_r \\ lnRa &= -a \cdot t_r + C \\ Ra &= e^{-a \cdot t_r + C} \\ Ra &= e^{C} \cdot e^{-a \cdot t_r} \end{aligned} \tag{2}$$

where a is the proportionality constant, C is the integration constant, C_{Ra} (C_{Ra}= is the constant having a value of the initial roughness in superfinishing (at tr=0) Ra_p. The equation (2) is replaced by

$$Ra = Ra_p \cdot e^{-a \cdot t_r} \tag{3}$$

If we will consider that the time of superfinishing is also proportional to the decrease of the roughness parameter Ra [Siketova 2009] [Trent 1979], we get the basic equation

$$-\frac{dRa}{dt_r} = (k_1 + k_2 t_r) Ra, \tag{4}$$

after the substitution of k_1, k_2 by proportionality constants a_1, a_2, we get the equation

$$Ra = C_{Ra} \cdot e^{-a_1 \cdot t_r - a_2 \cdot t_r^2}, \tag{5}$$

where C_{Ra}, a_1, a_2 are constants. Based on the fact that the pressure increase $\frac{dp_s}{dt_r}$ ıd the increase of the speed rati($\frac{dg}{dt_r}$ will be partially proportional to the decrease of roughness, one can write differential equation in the form

$$-\frac{dRa}{dt_r} = \left(a_1 + a_2 \frac{dp_s}{dt_r} + a_3 \frac{dg}{dt_r}\right) Ra \tag{6}$$

By the following modification one gets

$$\begin{aligned} lnRa &= -a_1 t_r - a_2 p_s - a_3 g + C \\ Ra &= C_{Ra} \cdot e^{-a_1 \cdot t_r - a_2 \cdot p_s - a_3 \cdot g} \ [\mu m] \end{aligned} \tag{7}$$

The relevant differential equation for our searching factors will be in the form

$$-\frac{dRa}{dt_r} = \left(\frac{a_1}{t_r} + \frac{a_2}{p_s}\frac{dp_s}{dt_r} + \frac{a_3}{g}\frac{dg}{dt_r}\right) Ra, \tag{8}$$

Its subsequent modification is the equation for the roughness Ra

$$Ra = C_{Ra} \cdot t_r^{-a_1} \cdot p_s^{-a_2} \cdot g^{-a_3} [\mu m] \tag{9}$$

Last equation is dimensionally non-homogeneous, but after the completion of etalon variables which are in the center of the schedule of the experiment plan [Lipa 2012], we

$$Ra = C'_{Ra} \cdot \left(\frac{t_r}{t_{ro}}\right)^{-a_1} \cdot \left(\frac{p_s}{p_{so}}\right)^{-a_2} \cdot \left(\frac{g}{g_o}\right)^{-a_3}, [\mu m] \tag{10}$$

where t_{ro}, p_{so}, g_o are etalon variables.

The equations (9) and (10) have the disadvantage that they are not valid for the initial conditions t_r=0, p_s=0, g=0. Therefore for the determination of the roughness parameter Ra, the differential equation of the increase is used

$$Ra = (Ra_p - Ra_m) e^{-a t_r} + Ra_m, \ [\mu m] \tag{11}$$

where Ra_p is the input value of Ra before superfinishing, Ra_m is the limit value of Ra achievable by superfinishing and a is constant. This equation one can extend to the following form

$$Ra = (Ra_p - Ra_m) e^{-a \cdot t_r - b p_s - c g} + Ra_m, \ [\mu m] \tag{12}$$

where b, c are constants.

3. Methodology of the original schedule of the experiment for the determination of constants for different fundamental equations for Ra.

The methodology concerns for the schedule of the linear model

$$v = a_o + a_1u_1 + a_2u_2 + a_3u_3 \tag{13}$$

The quadratic dependences one can change to linear models by the transformations. For the transformation of the linear model to required formulae, we again use the transformation equations. These equations we substitute into the linear model (13).

Transformation No.1

$$v = lny, u_1 = z_1, u_2 = z_2, u_3 = z_3 \tag{14}$$

one can get the exponential formula

$$lny = a_o + a_1z_1 + a_2z_2 + a_3z_3$$
$$y = e^{a_o+a_1z_1+a_2z_2+a_3z_3} \tag{15}$$
$$y = Ce^{a_1z_1+a_2z_2+a_3z_3}$$

Transformation No.2

$$v = lny, u_1 = lnz_1,\ u_2 = lnz_2, u_3 = lnz_3 \tag{16}$$

one can get the power equation

$$y = C \cdot {z_1}^{a_1} \cdot {z_2}^{a_2} \cdot {z_3}^{a_3} \tag{17}$$

Transformation No.3

$$v = \frac{1}{y}, u_1 = z_1,\ u_2 = z_2, u_3 = z_3 \tag{18}$$

one can get

$$y = \frac{1}{a_o+a_1z_1+a_2z_2+a_3z_3} \tag{19}$$

Transformation No.4

$$v = \frac{1}{y}, u_1 = \frac{1}{z_1},\ u_2 = \frac{1}{z_2}, u_3 = \frac{1}{z_3} \tag{20}$$

one can get

$$y = \frac{z_1 \cdot z_2 \cdot z_3}{a_oz_1z_2z_3+a_1z_2z_3+a_2z_1z_3+a_3z_1z_2} \tag{21}$$

Transformation No.5

$$v = y, u_1 = lnz_1,\ u_2 = lnz_2, u_3 = lnz_3 \tag{22}$$

one can get logarithmic equation

$$y = a_o + a_1lnz_1 + a_2lnz_2 + a_3lnz_3 \tag{23}$$

Transformation No.6

$$v = y, u_1 = \frac{1}{z_1}, u_2 = \frac{1}{z_2}, u_3 = \frac{1}{z_3} \tag{24}$$

one can get

$$y = a_o + a_1z_1 + a_2z_2 + a_3z_3 \tag{25}$$

Transformation No.7

$$v = \frac{1}{y}, u_1 = lnz_1,\ u_2 = lnz_2, u_3 = lnz_3 \tag{26}$$

one can get

$$y = \frac{1}{a_o+a_1lnz_1+a_2lnz_2+a_3lnz_3} \tag{27}$$

Transformation No.8

$$v = lny, u_1 = \frac{1}{z_1}, u_2 = \frac{1}{z_2}, u_3 = \frac{1}{z_3} \tag{28}$$

one can get

$$y = e^{a_o+\frac{a_1}{z_1}+\frac{a_2}{z_2}+\frac{a_3}{z_3}} \tag{29}$$

Transformation No.9 – in the case, that the angle of a rotation of the workpiece is equivalent to the involved quantities

$$v = y, u_1 = tgz_1,\ u_2 = tgz_2, u_3 = tgz_3 \tag{30}$$

one can get

$$y = a_o + a_1tgz_1 + a_2tgz_2 + a_3tgz_3 \tag{31}$$

Transformation No.10

$$v = tgy, u_1 = z_1,\ u_2 = z_2, u_3 = z_3 \tag{32}$$

one can get

$$y = arctg(a_o + a_1z_1 + a_2z_2 + a_3z_3) \tag{33}$$

The searched quantity: $y = Ra$

The factors: $z_1 = t_r,\ z_2 = p_s,\ z_3 = g$

For the help with the calculations in the planned experiment the coded factors are used in the basic equations.

$$x_1 = \frac{2(z_1-z_{1max})}{z_{1max}-z_{1min}} + 1,\ x_2 = \frac{2(z_2-z_{2max})}{z_{2max}-z_{2min}} + 1,\ x_3 = \frac{2(z_3-z_{3max})}{z_{3max}-z_{3min}} + 1 \tag{34}$$

Rationalized basic equations contain besides the coded factors also the etalon values from the center of the experiment plan (z_{10}, z_{20}, z_{30})

$$x_1 = \frac{2\left(\frac{z_1}{z_{10}}-\frac{z_{1max}}{z_{10}}\right)}{\frac{z_{1max}}{z_{10}}-\frac{z_{1min}}{z_{10}}} + 1,\ x_2 = \frac{2\left(\frac{z_2}{z_{20}}-\frac{z_{2max}}{z_{20}}\right)}{\frac{z_{2max}}{z_{20}}-\frac{z_{2min}}{z_{20}}} + 1,\ x_3 = \frac{2\left(\frac{z_3}{z_{30}}-\frac{z_{3max}}{z_{30}}\right)}{\frac{z_{3max}}{z_{30}}-\frac{z_{3min}}{z_{30}}} + 1 \tag{35}$$

For obtaining of further coded factors we use the transformation equations. The example of the obtaining of coded factors using transformation equations (16) is as follows

$$x_1 = \frac{2(lnz_1-lnz_{1max})}{lnz_{1max}-lnz_{1min}} + 1,\ x_2 = \frac{2(lnz_2-lnz_{2max})}{lnz_{2max}-lnz_{2min}} + 1, x_3 = \frac{2(lnz_3-lnz_{3max})}{lnz_{3max}-lnz_{3min}} + 1 \tag{36}$$

If we want the dimensional homogeneous equations, we must rationalize the coded factors according to the equations (35).

For the resulting equation of the parameter Ra we need to use regression equation in the form

$$y = b_o + b_1x_1 + b_2x_2 + b_3x_3\ [-] \tag{37}$$

where are regression coeficients, are etalon factors.

After achievement of the coded factors we can compile the relevant table with the help of planning of experiments. On the basis of three factors we choose three-factor experiment with the number of observations N=8. For each factor we choose the lower level $Z_{1\,min}$, $Z_{2\,min}$, $Z_{3\,min}$, the upper level $Z_{1\,max}$, $Z_{2\,max}$, $Z_{3\,max}$, and for the rationalized equations or for the testing j the medium level Z_{10}, Z_{20}, Z_{30}. The table has the form

Number of bservations i	z_1 resp. $\frac{z_1}{z_{10}}$	$\frac{1}{z_1}$	$\ln z_1$	$tg z_1$	z_2 resp. $\frac{z_2}{z_{20}}$	$\frac{1}{z_2}$	$\ln z_2$	$tg z_2$	z_3 resp. $\frac{z_3}{z_{30}}$	$\frac{1}{z_3}$	$\ln z_3$	$tg z_3$
1-8												
$\sum N = 8$												

i	y	$\frac{1}{y}$	$\ln y$	$tg y$	x_0	x_1	x_2	x_3	$x_1; y_i$	$x_2; y_i$	$x_3; y_i$
1-8											
$\sum N = 8$											

i	$x_1; \frac{1}{y_1}$	$x_2; \frac{1}{y_1}$	$x_3; \frac{1}{y_1}$	$x_1; \ln y_i$	$x_2; \ln y_i$	$x_3; \ln y_i$	$x_1; tg y_i$	$x_2; tg y_i$	$x_3; tg y_i$
1-8									
$\sum N = 8$									

Table 1. *Three-factor experiment – first part, second part and third part*

After the filling Table 1, we calculate column totals in required columns and we calculate the regression coefficients according the formula

$$b_i = \frac{1}{N} \sum x_{i\,;}\, y_i \; [-] \qquad (38)$$

Into the regression equation (37) we substitute regression coefficients and coded factors according to equation (34). By its following treatment we get the desired equation.

Conclusion

The evaluation of experimental data, it is sometimes very difficult. In search of mathematical formulas depending on the wanted values of input factors used linear model. Often it is not possible to use directly. Therefore, it is necessary to make a non-linear transformation of the linear model.

This article analyzes the possible shapes of equations for the mean arithmetic deviation of profile surface roughness in superfinishing a number of new relationships for surface roughness parameter Ra. These were created by applying various transformations. It can be used for experimental determination of mathematical formulas to calculate the surface roughness Ra parameter, using the theoretical background and method of experiment planning.

The article brings original (unpublished) results. The methodology was designed for superfinishing technology. However, it can be easily applied to determine the mathematical relationship of parameter Ra input factors for other machining technology. It is also possible to apply the methodology for determining the mathematical dependence of cutting force, cutting temperature and other parameters of the input factors in a variety of machining.

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