AN EFFICIENT ITERATIVE METHOD TO SIMULATE THE VARYING STATIC RECEPTANCE OF A THIN-WALLED WORKPIECE

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This paper introduces an accelerated iterative Conjugate Gradient method (CG-method) for simulating the varying static receptance of a thin-walled workpiece. The proposed approach enhances computational efficiency by computing an optimal initial displacement guess for a given cutter location. This guess is based on the previously calculated displacement vector from the preceding cutter location, serving as the start iteration vector for the CG-method. The validation of efficiency gains through this method is demonstrated by a significant reduction in computation time for a numerical example. Moreover, to ensure the accuracy of the accelerated iterative CG-method, comparisons are made with the Cholesky method. The results confirm the precision of the proposed approach.

KEYWORDS

thin-walled workpiece, static receptance, Cholesky decomposition, Conjugate Gradient method, initial displacement guess, finite element simulation

1 INTRODUCTION

The CAM planning for thin-walled workpieces is a significant challenge [Koike 2013] [Wiederkehr 2013]. A challenging issue in this context is the occurrence of shape errors due to the large static receptance of the thin-walled workpiece [Altintas 2018] [Denkena 2007] [Dittrich 2019]. Traditionally, refining CAM planning involves numerous machining tests, with iterative process adjustments until suitable process parameters are determined [Bolar 2016]. Unfortunately, this approach results in considerable material wastage and time consumption [Bolar 2016].

A potential solution lies in using finite element simulation to predict the static displacement of thin-walled workpieces, and enable the compensation of shape errors based on these predictions [Li 2018] [Ratchev 2005] [Wiederkehr 2013] [Wimmer 2019]. However, simulating thin-walled workpieces via finite element methods presents unique challenges due to the varying nature of the system stiffness matrix evolving with material removal. Furthermore, the process force fluctuates across different cutter locations. Because of the varying static stiffness and the fluctuating process force, multiple simulations are needed to calculate the varying workpiece displacement for one machining process. This leads to high computation costs, limiting the usage of the simulation methods to predict the displacement of thin-walled workpieces.

To reduce the computation cost, a novel method is introduced for simulating the varying static receptance of thin-walled workpieces in previous work [Brecher 2023]. This method is based on the Cholesky method employing Cholesky decomposition [Bathe 2014] [Scott 2023]. By reusing the Cholesky factors, the novel method offers a more efficient solution to calculate the varying static stiffness of the intermediate states of a thin-walled workpiece. As shown in Fig. 1, the calculation of the static receptance at a single cutter location using this novel method consists of the following steps [Brecher 2023]:

1. Detect the removed elements based on the cutter location, and update the system stiffness matrix of the workpiece.
2. Identify the nodes in the search circle of the radius r around the tool center point.
3. Define a concentrated static force $f_{concentrated}$ with an arbitrary amplitude in any orthogonal direction.
4. Distribute the concentrated force to multiple identical forces $f_{distributed}$ on the nodes found at the first step.
5. Solve Eq. 1 using a direct method based on Cholesky-decomposition:

$$K \cdot u = r \qquad (1)$$

with:

K: system stiffness matrix

u: static displacement vector

r: the load defined in step 3 expressed as column vector

The Cholesky factor of the last cutter location is reused to reduce the computation time of u.

6. Extract the displacements of the nodes identified in the second step and compute the average displacement.
7. Compute the static receptance by

$$g_{i,j} = \frac{u_j}{f_i} \qquad (2)$$

with:

i: direction of the force

j: direction of the displacement

$g_{i,j}$: static receptance

u_j: average displacement in direction j

f_i: concentrated force in direction i

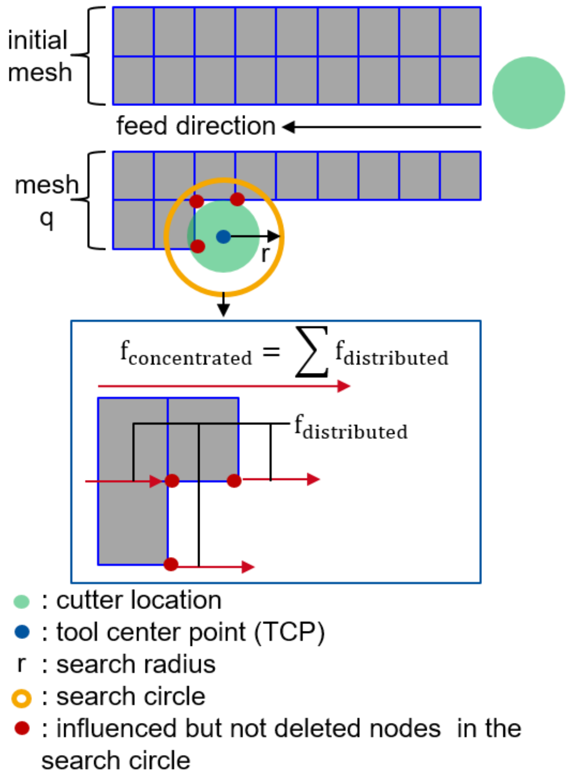

Figure 1. Computation of the static receptance by using Cholesky method

It's important to note, that solving eq. 1 requires the time-consuming Cholesky decomposition of stiffness matrix K, which varies at each cutter location due to material removal. To reduce the time cost of step 5, the Cholesky factor of the previous cutter location is partly reused. However, this step is still the most time-consuming part of the process [Brecher 2023]. An alternative to compute the displacement vector in step 5 is the iterative conjugate gradient method (CG-method). The CG-method consists of the following steps [Bathe 2014] [Hackbusch 2016] [Hestenes 1952]:

- Choose a start iteration vector u^0 (initial guess, u^0 is typically a null vector).
- Calculate the residual:

$$e^0 = r - Ku^0 \tag{3}$$

- If the vector norm $\|e^0\|$ is smaller than the convergence tolerance ε:
    - The start iteration vector u^0 is the solution, and the iteration process is stopped.
- Else:
    - Set $p^0 = e^0$.
    - Calculate for $s = 0, 1, 2, \ldots$,

$$\alpha_s = \frac{(e^s)^T e^s}{(p^s)^T K p^s} \tag{4}$$

$$u^{s+1} = u^s + \alpha_s p^s \tag{5}$$

$$e^{s+1} = e^s - \alpha_s K p^s \tag{6}$$

$$\beta_s = \frac{(e^{s+1})^T e^{s+1}}{(e^s)^T e^s} \tag{7}$$

$$p^{s+1} = e^{s+1} + \beta_s p^s \tag{8}$$

The for loop between Eq. 4 and Eq. 8 is a process to iteratively update u. This update process continues until the vector norm $\|e^{s+1}\| < \varepsilon$, signifying the fulfillment of the convergence criterion.

The CG-method's efficiency is tied to its convergence rate, having a direct impact on computation time. The inherent drawback of the CG-method lies in the possibility for slow convergence, leading to high computation time. However, it's important to note that this drawback is counterbalanced by the method's ability to be highly efficient when convergence occurs rapidly. In instances where the CG-method converges rapidly, it can markedly outperform the Cholesky-method, resulting in significantly reduced computation time.

In summary, during the machining process, material removal causes the system stiffness matrix of the workpiece to vary. This variation can be accounted for by subtracting the elemental stiffness matrix of the removed material from the initial system stiffness matrix [Brecher 2023]. As a result, the static stiffness changes at each cutter location. The unique challenge is to solve the varying static stiffness at each cutter location efficiently and accurately.

This paper primarily seeks to enhance the convergence rate of the CG-method in simulating thin-walled workpieces. Following this introductory section, section 2 outlines a strategy for augmenting the convergence rate by generating an effective initial guess within the CG-method. Subsequently, section 3 systematically evaluates both the efficiency and accuracy of the CG-method. The concluding section encapsulates key findings and outlines future perspectives.

2 ACCELERATION OF THE CG-METHOD BY USING A GOOD INITIAL GUESS

Examining Eq. 3, it is observed that the CG-method achieves convergence in a single iteration if the initial guess is identical to the real displacement vector. This observation leads to the following hypothesis:

- Hypothesis 1: The convergence rate of the CG-method can be increased by minimizing the disparity between the initial guess and the accurate displacement vector.

Assuming the validity of this hypothesis, the convergence rate of the CG-method to simulate an intermediate state q of a thin-walled workpiece can be increased by using the following process:

- Generate an initial guess u_q^0 by selecting entries from u_{q-1}. The chosen entries correspond to the common DoF of meshes q and q-1.
- Solve the displacement vector u_q using the generated initial guess u_q^0

This process is illustrated in Fig. 2, where each node has a DoF in horizontal direction. Mesh q-1 comprises 20 nodes and 20 corresponding DoF. The displacement vector $u_{q-1} \in \mathbb{R}^{20\times1}$ is known from a prior simulation. Mesh q is derived by removing element E1 from the mesh q-1, resulting in the deletion of nodes N1 and N20. The entries in u_{q-1} corresponding to the remaining nodes are selected to generate an initial guess $u_q^0 \in \mathbb{R}^{18\times1}$ for mesh q. This initial guess is employed as the start iteration vector to calculate $u_q \in \mathbb{R}^{18\times1}$. Given the small difference between meshes q and q-1, along with similar external loads, u_q^0 is expected to be a reliable approximation of the accurate displacement vector u_q, facilitating a high convergence rate for the CG-method in mesh q.

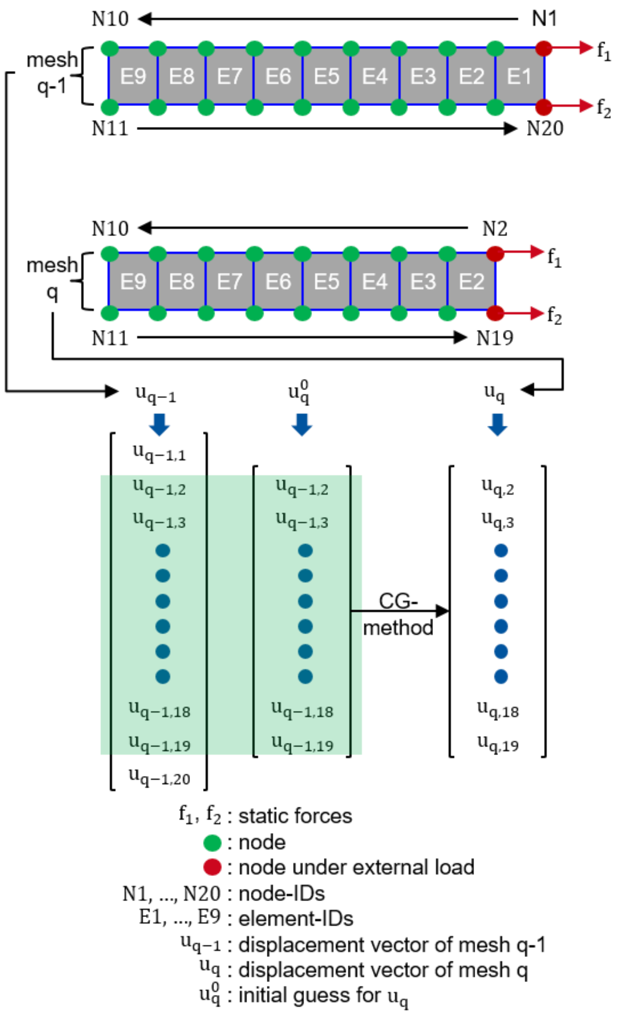

Figure 2 : Generation of an optimal initial guess for CG-method

It's crucial to acknowledge that the method for generating a good initial guess for mesh q utilizes the calculated displacement vector of the previous mesh as input. Consequently, this method is not applicable to the first mesh. Addressing this special case for the first mesh and the normal case for the other meshes, the full process to calculate displacement vectors for all intermediate meshes of a thin-walled workpiece involves the following steps:

- Calculate the displacement vector u_1 for $q = 1$ (first mesh) with a null vector as the initial guess, using standard CG-method.
- For $q = 2, \dots, Q-1, Q$, where Q is the total number of the intermediate meshes:
    - Update the system stiffness matrix by subtracting the elemental stiffness matrix of the removed material from the initial system stiffness matrix. The size of the system stiffness matrix is not limited.
    - Generate the initial guess u_q^0 based on u_{q-1}.
    - Calculate u_q using CG-method with the generated initial guess u_q^0.

The presented section introduces a concise approach to generating a proficient initial guess for the CG-method in simulating thin-walled workpieces. The subsequent section evaluates the efficiency and accuracy of the CG-method when employing this well-generated initial guess, demonstrated through a numerical example.

3 NUMERICAL TEST

In this section, the method introduced in the previous section is evaluated through a numerical example involving the flank milling of a thin-walled workpiece, as illustrated in Fig. 3. The initial mesh of this workpiece comprises 236,715 tetrahedron elements and 47,489 nodes. This specific numerical example has been employed in prior research to validate the novel Cholesky method [Brecher 2023]. In the prior research, the removed finite elements were detected for 47 cutter locations (CLs). These 47 CLs are uniformly distributed along the tool path, with a consistent spacing of 2 mm between any two adjacent CLs. Furthermore, the system stiffness matrix is updated for these

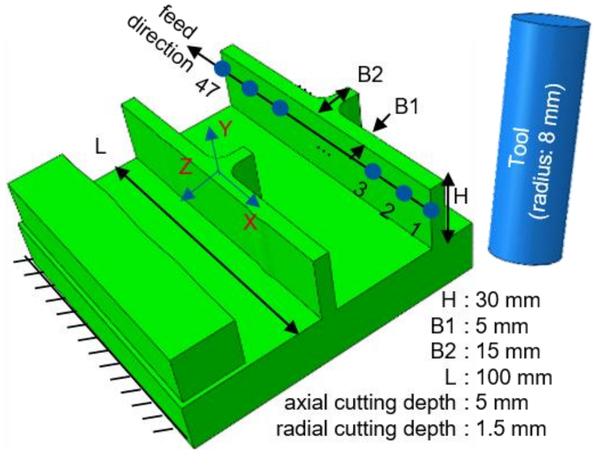

Figure 3. Numerical example for testing the accelerated CG-method

CLs by subtracting the elemental stiffness matrices of the removed elements from the initial system stiffness matrix. The static receptance at these 47 CLs is calculated using an accelerated Cholesky method [Brecher 2023]. To facilitate a meaningful comparison, the static receptance is computed for these 47 cutter locations (CLs) using the newly proposed accelerated CG-method outlined in the preceding section.

The computation times for the receptance at each CL are detailed in Fig. 4.

Utilizing the CG-method with a well-generated initial guess based on the displacement vector of the previous CL results in an average computation time of 8.2 s. In contrast, without employing the good initial guess, the average computation time with the CG-method rises is 16 s. Moreover, the average computation time of 8.2 s is significantly shorter than the average computation times of 33.3 s and 63.1 s when employing the Cholesky-method, irrespective of reusing or not reusing the Cholesky-factor. This difference underscores a substantial efficiency enhancement achieved by employing the CG-method with an initial guess generated by the method introduced in the previous section. The significant efficiency improvement observed confirms the hypothesis presented in the last section.

The accuracy of the novel Cholesky-method has already been validated in prior research through machining test and simulation with commercial software [Brecher 2023], making the static receptance computed by the Cholesky-method a reliable reference. Compared to the Cholesky-method, the CG-method is an iterative method. As a result, the CG-method can have the same accuracy as the Cholesky-method, only when the

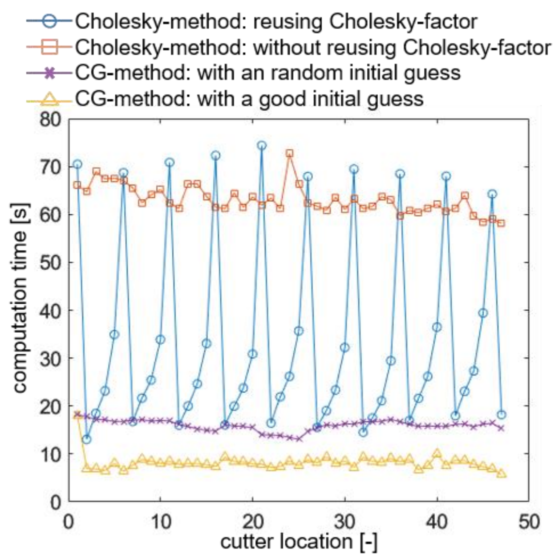

Figure 4: Time to compute the varying static receptance of the thin-walled workpiece

convergence tolerance ε (explained in introduction) is equal 0. However, the convergence tolerance ε is typically larger than 0, in order to have a high convergence rate. In this example, the convergence tolerance ε is set to 0.01. Hence, it becomes necessary to assess the accuracy of the CG-method by comparing the calculated static receptances using both the CG-method and the Cholesky-method. This comparison is illustrated in Fig. 5. The errors of the CG-method for all the CLs fall within a small range between -0.5 % and 0.7 %. This proves the accuracy of the CG-method.

4 CONCLUSIONS

This paper introduces a concise and efficient approach for simulating the varying static receptance of thin-walled workpieces, using an accelerated iterative CG-method. The method's efficiency is notably enhanced by generating a good initial guess based on the simulated displacement vector of the preceding CL. Comparative analysis demonstrates its outstanding efficiency compared to the Cholesky-method, while maintaining the same level of accuracy. The combination of high efficiency and good accuracy of this novel approach makes it well-suited to optimize the machining processes of thin-walled workpieces.

In future work, the method will be applied to real workpieces during CAM planning to validate its efficiency and accuracy under practical conditions. Furthermore, an efficient method for predicting the varying dynamic characteristics of thin-walled workpieces will be researched.

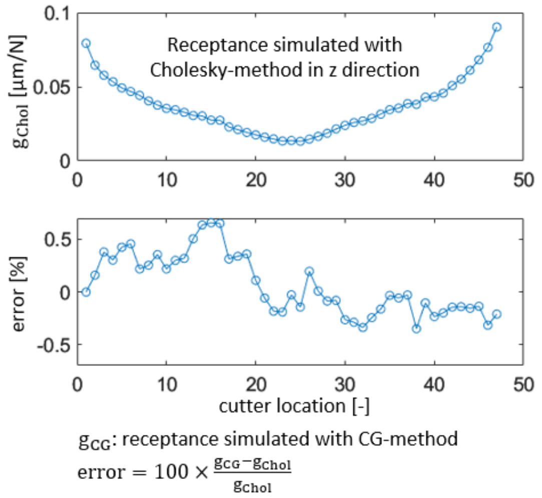

Figure 5: Error of the CG-method for computing the varying static receptance of the thin-walled workpiece

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