The multiple motor drives are complex and coupled MIMO nonlinear systems. Due to the complexity of their mathematical models, the development of effective control systems is quite a complicated task. This paper presents the design of optimal control based on the quadratic optimality criterion for the central section of the continuous production line using soft computer methods when fuzzy models of the controlled system have been used. In this case, the proposed method demonstrates the non-analytical design of the optimal parameters of PI controller with emphasis on minimal knowledge on the controlled system. The realized experimental measurements on a continuous line laboratory model confirmed the effectiveness and the good dynamic properties of the optimal continuous line controller and also its applicability to other MIMO nonlinear dynamic systems.

**KEYWORDS**
Continuous line, fuzzy model, MIMO nonlinear systems, optimal control, soft computing

1 INTRODUCTION

The term ‘multi-motor drive’ describes all the drives in a technological process. A typical representative of a multi-motor drive is the continuous line (CL), where the individual working machines are coupled by each other through the material. They are, e.g., lines for processing continuous flows of material (e.g., sheet metal strips, tubes, processing lines in paper mills and printing works, etc.) by material tension in the form of tension, while the web moves at a defined line speed. In principle, we are dealing with a nonlinear MIMO system in which there is strong interaction between the individual input and state quantities which can result in bad quality or even destruction of the material being processed. For this reason, it is not quite possible to apply known methods from the area of SISO systems in the synthesis of its control. These methods may secure invariance of the CL against defects, however, they will not secure (especially in dynamical states) the desired autonomy of control of individual output quantities; for example, a quick change of desired line speed may result in deformation or break of the web [Jeftenic 2006].

Many different control methods applied to web tension control problems have been proposed during the past decades. The web tension control problems review can be found in [Wolfenmann 1995]. Many industrial web-transport systems have used PID or PI type controllers [Allaoua 2010, Bouchiba 2012]. These control methods are simple but the coupling between tension and speed limit their performance. Robust controllers could guarantee robust stability considering disturbance and uncertainty. However, robustness is ensured only for a small range of parameter changes [Baumgart 2003, Paglia 2000]. The advanced control methods, such as observed based feedback control [Song 2000, Lin 2003], time optimal control [Angermann 2000] and $H_{\infty}$ [Knittel 2003] are too complex to implement and not well understood by industry.

In recent years, also the fuzzy systems have been utilized in different structures for designing efficient controllers for different MIMO systems [Okada 1998, He 2012, Fedor 2016, Li 2016]. The disadvantage is that the performance of fuzzy controllers often leads to problems concerning control dynamics and control stability. These facts make the controller design methods and their resulting structures very complex which prevents an obstacle to their wider practical application in industry. The increasing complexity of dynamical systems such as this multi-motor drive with the mechanical coupling of the drives requires very often the use of optimal solutions and easy applicable control approaches.

In the research discussed here, the design of optimal continuous line control in terms of the selected optimization criterion is introduced. The optimal controller design method consists in the development of the algorithm for identifying the optimal parameters of the PI type controller within the chosen range of possible input and output values. The used method requires knowledge of a high-quality model of the controlled system. Most often, they are analytical models. In our case, the fuzzy model of the central section of the continuous line proposed by the authors in [Perdukova 2023] was used. The aim was to demonstrate the application of this CL fuzzy model in non-analytical, fully computer-oriented design of a continuous line optimal PI controller involving minimal demand for detailed data on the controlled technology. The properties of the optimal PI controller were verified by experimental measurements on a laboratory model of the line built at the workplace of the authors.

2 CONTINUOUS LINE ANALYSIS

In industrial practice, there exist many various typical multi-motor drive configurations [Zhao 2017] where the tension in the web arises due to different circumferential speeds of the web rolls, partially due to the difference of their positions. These lines usually consist of three autonomous sections:

- The entry section consists of unwinding machine for accumulating a stock of material for the technological section and for reduction of tension in the web.
- The technological section, where technological operations are carried out according to the technological prescription for the particular material.
- The exit section consists of winding machine for coiling of the web of materials.

For simplicity only the coupling of two machines (central section of continuous line) is investigated but this idea can be extended to an indefinite number of machines coupled by processed material. Figure 1 shows the structure of the central section of the continuous line. The structure consists of two DC motors supplied by power electronic converters TC with input voltages $v_1$, $v_2$. The work machines of the line are driven by these motors through the gear with the gear ratio $j$; quantities $v_1$, $v_2$ are machine rolls circumferential speeds, quantity $F_{12}$ is the tension...
in the web of material between the two machines. \(K_v\) is circumferential speed sensor (considering \(v = r\omega\), where \(r\) is roll radius and \(\omega\) is motor angular speed), \(K_t\) is tension sensor, \(u_{v1}\), \(u_{v2}\) are output voltages from the speed sensors and \(u_{t1}\) is the output voltage of the tension sensor.

The main line disturbances are tensions before \((F_{01})\) and after \((F_{03})\) the central section of the considered line which affect the load torques of the first and of the second drives.

The laboratory model presents a functional model of a multimotor drive shown in Fig. 3 where the drives are mutually coupled by a running web of elastic material which is a magnetic tape of the 0.03 m width. The web, wound up into a coil, runs from the un-coiler to coiler, while it is belt round three work rolls (used for a better friction between the web and roll surface).

The model is driven by 5 DC disc motors powered by Allen Bradley DC converters 1386 DC Servo Drive System with PWM modulation. The control system is based on a programmable controller PLC S7-400 with FM458 technological card. CFC language is used for the control program development. Controlling voltages for the converters within the range of \(\pm 10\) V present inputs into the system, and the speeds of the drives and tensions in the sections between the work rolls present outputs from the system. Incremental sensors (IRC) generating 4 000 increments per revolution are used for measuring the revolutions of the motors. The tensions are sensed by two tension sensors. In our experiment to demonstrate the proposed control method we utilise the section with only two rolls and one tension sensor. The laboratory model of the continuous line is shown in Fig. 4. Its parameters are specified in the Appendix.

To determine basic properties of CL, experimental identification measurements were performed on the laboratory model of the CL for current pulses \((u_{v1} = i^*_1\) and \(u_{v2} = i^*_2\)) applied sequentially to each input of the model. In Fig. 5 the time responses of the CL outputs are shown, i.e., \(y_1 = u_{v2}\) and \(y_2 = u_{v2}\).

3 LABORATORY MODEL SETUP

Obtained results were verified by experimental measurements on a laboratory model of a continuous line that was built in the laboratory of the authors.
From the Fig. 5 it is obvious that the system contains a fast (tension) subsystem with oscillating response and a slow (speed) subsystem. They are coupled and mutually interact.

4 CONTINUOUS LINE DESCRIPTION

This system with the mechanical coupling of two machines presents a 3rd order nonlinear MIMO system with two inputs and two outputs (see Fig. 6 and Fig. 2). The parameters of the system change depending on the mechanical properties of the material and on the speed of its motion.

Figure 6. Inputs, outputs and states of the continuous line modelled as MIMO system

General form of the state-space description of the dynamical system is:

\[
\frac{dx}{dt} = Ax + Bu + Ef,
\]

\[
y = Cx + Du
\]

where vector \( f \) represents disturbances \( f = [F_{u1}, F_{s2}]^T \).

If we choose the inputs, outputs and states of the CL system according to Fig. 6, then the matrices of the state-space description have the form

\[
A = \begin{bmatrix}
0 & -\frac{r^2}{J_{p1}} & \frac{r^2}{J_{p1}} \\
\frac{K_{d1}^2}{J_{p1}} & 0 & \frac{K_{d1}^2}{J_{p1}} \\
0 & -\frac{r^2}{J_{p2}} & 0
\end{bmatrix},
B = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
c_b r & 0 & 0
\end{bmatrix},
C = \begin{bmatrix}
0 & K_T & 0 \\
0 & 0 & K_d
\end{bmatrix},
D = \begin{bmatrix}
0 & 0
\end{bmatrix},
\]

For the computer-oriented design of the optimal controller, we can use the description of the CL system in the form of a block diagram in Fig. 2, which corresponds to the state description in Eq. 1-3.

However, some parameters of this description cannot be precisely determined, for example those related to material properties (parameter \( SE \), damping constant \( K_t \)). For this reason, a non-parametric fuzzy model of a continuous production line with emphasis on minimal knowledge on the modelled system was proposed by the authors in [Perdoková 2023]. The fuzzy model structure is based on the state space representation of the dynamic system in discrete form according to which the state of a system in a particular step depends on its state in the previous step and on the increment in state between these steps. Considering the choice of CL input, state and output quantities according to Fig. 6, the structure of the CL fuzzy model is shown in Fig. 7. To determine the output of the tension fuzzy model \( \dot{y}_1 = \dot{x}_2 \) and the output of the speed fuzzy model \( \dot{y}_2 = \dot{x}_3 \) in the \( k \)-th step, it is necessary to measure the values of the inputs \( u_1, u_2 \) and state variables \( x_1, x_2, x_3 \) in the \( k-1 \)-th step.

The first step in the design of the fuzzy model for the central section of the continuous line is the establishment of a consistent database from measured inputs and their corresponding outputs, which covers its entire assumed work-space. Then the measured database is used for searching two FIS structures of the given nonlinear system which best describe the measured relations between inputs and outputs, i.e. \([u_{1k-1}, u_{2k-1}, x_{1k-1}, x_{2k-1}, x_{3k-1}] \rightarrow \dot{y}_{1k}\) and \([u_{1k-1}, u_{2k-1}, x_{1k-1}, x_{2k-1}, x_{3k-1}] \rightarrow \dot{y}_{2k}\).

Figure 7. Continuous line fuzzy model structure

Using the measured database, the fuzzy models for tension \( y_1 = x_2 \) and speed \( y_2 = x_3 \) subsystems have been designed by procedures of cluster analysis in MATLAB (ANFIS tool).

The results were two static Sugeno type fuzzy systems with two rules for each output quantity. The CL fuzzy model properties were demonstrated by comparing its outputs with those of the laboratory model for randomly generated input signals \((u_1, u_2)\) different from the inputs for which the fuzzy model database was created. The obtained results have confirmed that the CL fuzzy model is of very high quality and accuracy approximating very well the performance of the continuous line laboratory model as shown in Fig. 8.

Figure 8. Comparison of fuzzy model and real laboratory model outputs \( y_1 = u_{1k} \) and \( y_2 = u_{2k} \) for a continuous line

5 OPTIMAL CONTROLLER DESIGN FOR CENTRAL SECTION OF A CONTINUOUS LINE

From the point of view of production technology, the principal aim of CL control consists in achieving good dynamic control of tension in the material, with the speed of the moving material corresponding with the pre-set CL speed.

The system’s control objectives are the following:
- Optimal dynamics during the determined operational cycle for the selected criterion.
- Autonomous control of CL variables, such as speed and tension, i.e. system decoupling.
- Invariance against additive disturbances.
- Robustness against CL parameter changes which can be caused by changes of elastic coupling properties between the working machines.

Processing of material in a CL is usually carried out in operation cycles during which a required amount of prepared material is processed (e.g., a roll of paper, a sheet metal coil.) An operation
cycle includes three stages - line start-up, line running at constant processing speed, and line delayed shut-off.

To meet requirements of the control goals of central section of CL (for which the fuzzy model was designed) in terms of the selected optimality criterion we chose the simplest control structure consisting of two standard PI controllers (one for tension control \(y_1= u_{12}\) and one for speed control \(y_2= u_{2}\)), as shown in Fig. 9.

The optimization algorithm works as follows: at the beginning of the optimization, we chose the initial values of the controller parameters \(K_{PF0}=K_{PF}=K_{PV}=K_{PF0}=1\) (vector \(\mathbf{k}_0\)) and the space of parameters of particular controller gains is divided by the increments \(\Delta K_P=1, \Delta K_P=5,\Delta K_P=1, \Delta K_P=5\). Maximum values for each parameter are defined: \(K_{PFmax}=10, K_{PFmax}=100, K_{PFmax}=10, K_{PFmax}=100\), according to the usual values of the proportional and integration components of the PI controllers. The criteria function for initial values of the vector \(\mathbf{k}_0\) and for the selected operation cycle of CL has a value \(J(\mathbf{k}_0) = 37.19\). The time responses of CL laboratory model output quantities corresponding to this initial PI controller parameters setting are shown in Fig. 11.

Note: The quantities in the following graphs are normalized in respect to rated values.

For such control structure, optimal parameters of the controllers were searched. The aim of the optimization consists in finding vector of the controller parameters \(\mathbf{k} = [K_{P1}, K_{I1}, K_{P2}, K_{I2}]\) so that the selected optimization criterion for a given CL operation cycle is minimal. The optimization criterion was chosen in the quadratic form:

\[
J(\mathbf{k}) = \int (C_1 e_1^2 + C_2 e_2^2) \, dt
\]  
(4)

where \(e_1\) and \(e_2\) are the control deviations of the tension \(u_{12}\) and the strip speed \(u_2\) from the reference values, and the coefficients \(C_1\) and \(C_2\) determine the weights (importance) that we confer to the individual output control deviations. In our case, we have chosen the values \(C_1 = 5\) and \(C_2 = 1\), which are physically interpreted as giving a stronger emphasis on quality of the tension control deviation \(e_1\) (i.e., to the tension in the material strip that primarily determines the quality of the output product). The optimal value of the parameters of the controller vector \(\mathbf{k}\) is to be searched in the space of real values of gains of proportional and integration components.

The method represents a search of an extreme of a function of more variables, for which several methods of optimization are available [e.g. genetic algorithms, network charts, etc.]. In our case, the method of uniform geometric partitioning of the parameter space into equal intervals and its systematic scan was chosen. This was realized by an m-file in MATLAB/Simulink environment according to Fig. 10. The advantage of this procedure is that the global minimum of criterial function (4) can be always found. The disadvantage consists in the time and computational complexity if the vector \(\mathbf{k}\) has more parameters and the division of the space is denser.

The optimization algorithm works as follows: at the beginning of the optimization, we chose the initial values of the controller parameters \(K_{PF0}=K_{PF}=K_{PV}=K_{PF0}=1\) (vector \(\mathbf{k}_0\)) and the space of parameters of particular controller gains is divided by the increments \(\Delta K_P=1, \Delta K_P=5,\Delta K_P=1, \Delta K_P=5\). Maximum values for each parameter are defined: \(K_{PFmax}=10, K_{PFmax}=100, K_{PFmax}=10, K_{PFmax}=100\), according to the usual values of the proportional and integration components of the PI controllers. The criteria function for initial values of the vector \(\mathbf{k}_0\) and for the selected operation cycle of CL has a value \(J(\mathbf{k}_0) = 37.19\). The time responses of CL laboratory model output quantities corresponding to this initial PI controller parameters setting are shown in Fig. 11.

Note: The quantities in the following graphs are normalized in respect to rated values.

When verifying the properties of the proposed controller, we have assumed that external (additive) disturbance could occur, i.e., a step change of the tension \(F_{12}\) before the considered CL section with the amplitude of rated tension \(F_{12}\) at time \(t = 20\) s. In real CL this disturbance can be caused by a sudden short change of the thickness in the material strip – when welding one strip ending to the beginning of other one.

After finding the optimal parameter vector \(\mathbf{k}_{opt} = [9, 20, 7, 80]^T\), we obtained the value of the optimization criterion \(J(\mathbf{k}_{opt}) = 0.3906\). Time responses of CL laboratory model output quantities for optimal controller parameters are shown in Fig. 12.

6 DISCUSSION

Figures 11 and 12 show experimental results of the CL central section control for selected operational cycle. In industrial practice the required tension in the strip of material is set the first and then the CL starts up to the desired operational speed. Figure 11 shows the selected CL operation cycle in which the desired value of tension in the strip of material is first set to \(u_{12}=0.8\) N and at time \(t = 4\) s the CL starts up to reach the operational speed. We can see that for initial values of parameters of CL speed and tension PI controllers the deviation in tension from the desired value is up to 10%, the speed...
deviation is up to 25%, and at certain moments the CL also runs in the opposite direction. Autonomy and invariance in term of disturbance is poor in dynamic states.

On the contrary, when we set optimal values of parameters of CL speed and tension PI controllers, we can see, as is shown in Fig. 12, that tension in the CL is in the range of 2% also in dynamic states (which ensures high material processing quality during the whole operation cycle), and CL speed only briefly falls outside the desired value by approx. 8% when the disturbance \( F_{01} \) is acting. Also, we can see that there is practically no mutual coupling between the CL tension and speed, whereby the requirement of autonomy for this MIMO system is met.

Due to the strong coupling between CL speed and tension during a normal operation cycle there exist many sources of disturbances, e.g., strip sliding along surface of the work roll or change of the material properties (i.e. the strip elasticity and damping). These disturbances influence CL tension and can lead to wrinkling or even breaking of the material. Therefore, robustness during the process is an important objective of the CL control strategy.

The robustness of the proposed control structure has been verified by changes of important parameters of the controlled system that significantly affect properties of the elastic strip, namely the damping of the processed strip material \( k_t \) (corresponding to changed material properties). Figure 13 shows the time responses of the CL speed and tension when the damping constant of the material was decreased five times \( k_t = 0.2k_t \) (i.e., five times more elastic material).

Similarly, Figure 14 shows the dynamics of the speed and tension control for the case when the damping of the strip material was increased five times \( k_t = 5k_t \) (i.e., five times less elastic material).

From the point of view of the real CL, there are significant and border changes in values of the parameter under consideration, while the dynamics, decoupling and invariance of the CL control remains almost unchanged. This confirms the robustness of the proposed controller.

Experimental measurements have confirmed that the proposed controller is able to meet the basic control objectives, i.e., optimal dynamics, invariance to external disturbance, robustness against changes of important parameters and that ensure high quality material processing throughout the entire operation cycle (including the transient states).

Optimal control of the CL is designed for a standardized operational cycle of a continuous line and is realized by classic continuous PI controllers. These controllers will be optimal until external disturbances in the line (in this case primarily the tension before and after the central section of the CL) do not exceed physically critical values, i.e. until the actuators work on their physical limitation (the static converters supplying the drives in the CL do not hit the current limitation). Outside this range, in principle, no control is possible that would ensure the defined maximum control error, and in this case the line is stopped due to uncontrollable error (emergency condition).

The quality of the proposed optimal controller depends on the quality of the nonlinear system’s description. Conventional methods of designing controllers for CL require its analytical description and precise knowledge of its parameters. The design of the controllers is more complex, and the resulting control structure is more complicated [Pin 2013, Shafiee 2014, Priva 2015]. Although the design methods of fuzzy PI controllers [Fedor 2013, Li 2016, Leso 2018, Khalifa 2022] do not require knowledge of the mathematical model of the controlled system, they are very demanding to obtain its qualitative properties (for example from experts). For the stated reasons, the optimal controller design method based on the fuzzy CL model was chosen in the article, which in principle allows to describe and investigate the behaviour of CL without exact knowledge of its structure and parameters. Compared to conventional methods and methods based on the use of fuzzy logic, the proposed method is simpler, allows the use of standard soft computing tools in the design and leads to high-quality dynamic responses, while the invariance and robustness of the controlled system is ensured. The advantage of this method is also that it implicitly guarantees the autonomy of the control of the tension and speed of CL, and its quality is defined by the criterion of optimality (Eq. 4).

Various optimality criteria or their combinations could be chosen according to the particular requirements related to dynamics, power efficiency or also optimization for the determined operational cycle of the CL. This is a major advantage from the practical point of view. For example, the selection of a suitable energetic optimality criterion can bring about marked energy savings.

**7 CONCLUSION**

The paper describes the method for the design of the optimal control based on the quadratic optimality criterion for the central section of a continuous line involving minimal demand for detailed data on the controlled technology.

In the design process an analytical description of the controlled system was substituted by its experimentally obtained fuzzy model designed only on the controlled system input/output relations. A very simple control structure with two PI controllers was proposed for the tension and speed control of the CL central section. Optimal parameters in this structure were searched, such that would best satisfy the chosen quadratic optimality criterion for the given operation cycle of the controlled system.
The properties of the proposed controller have been verified by experimental measurements on the laboratory model of the continuous line. The obtained results confirm advantages and correctness of the proposed PI control structure: it is simple, robust against changes of important parameters, invariant to operating disturbances, and ensures a high-quality dynamics of the controlled system during the whole operating cycle. Proposed controller is optimal within the chosen range of its parameters and can be designed without the knowledge about the analytical model of the controlled system using soft computing methods.

For stated reasons the proposed method of designing an optimal PI controller for a nonlinear dynamic system for which only external information is available can be considered as an enhancement to the wide range of fuzzy model based control methods, which, due to its simplicity and control quality, could find wide application in industrial practice.

With this method no principal limitations for the controlled system’s nonlinearities are defined, and therefore broad application of the presented method in industrial practice can be assumed, for example in multi-motor drives in steel industry, paper-making, printing and textile industries, in the production of synthetic fibres and foils in the chemical industry and so on.

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APPENDIX
Parameters of the Continuous Line Laboratory Model
DC motors and gears:

\[ P_R = 140 \text{ W} \]

\[ n_R = 3650 \text{ rpm} \]

\[ T_R = 0.37 \text{ Nm} \]

\[ U_R = 24 \text{ V} \]

\[ I_R = 8.5 \text{ A} \]

\[ J = 0.002 \text{ kgm}^2 \]

\[ R_0 = 0.7 \Omega \]

\[ L_0 = 0.1 \text{ mH} \]

\[ c = 0.043 \text{ Vs} \]

\[ j = 24 \]

Processed material:

\[ F_{3a} = 25 \text{ N} \]

\[ b = 0.03 \text{ m} \]

\[ h = 0.1 \times 10^{-3} \text{ m} \]

\[ S = 3 \times 10^6 \text{ m}^2 \]

\[ \varepsilon = 1.8 \times 10^4 \text{ Nm}^{-2} \]

\[ SE = 5400 \text{ N} \]

\[ l = 1.35 \text{ m} \]

Sensors:

\[ K_f = 0.4 \text{ V/N (tension sensor gain)} \]

\[ K_v = 16.6 \text{ V/ms}^{-1} \] (circumferential speed sensor gain)

\[ K_i = 2 \text{ V/A (current sensor gain)} \]

Work rolls:

\[ r = 0.04 \text{ m} \]

\[ \nu_{max} = \nu_{2max} = 0.6 \text{ m/s}^{-1} \]

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