THE INFLUENCE OF THE OPTIMIZATION OF THE SPUR GEAR BODY ON THE MESHING STIFFNESS

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The design of the body shape of spur gears has an effect on the deformation and thus also the stiffness of the gearing. The stiffness of the gearing is a parameter that significantly affects the noise level of transmission mechanisms. Determining the gear stiffness is difficult due to the shape of the gear teeth. The paper examines the influence of the shape and size of individual gear body parameters on gear deformation. Deformation is solved by finite element method. On the basis of the deformation of the gearing, the stiffness of the gearing is determined.

KEYWORDS

Deformation of gearing, meshing stiffness, gear body, weight, shape, FEM.

1 INTRODUCTION

Nowadays, ever higher demands are placed on machine parameters. This is manifested in the growth of the dynamic load, in the increase in the speed of individual parts [Wojnar 2021]. All this is realized with increasing demands on performance, precision, lifetime and reliability of the machines themselves. Therefore, it is necessary to consider the dynamics of the processes that take place in the machines.

Internal gearing dynamics is one of the most widespread gearing problems [Czech 2022]. It is manifested by the vibrations of all parts of the gears, their noise and increased stress on the teeth. These vibrations in gearboxes manifest themselves as a significant source of noise [Duhancik 2024]. There are many influences that cause vibrations in gearboxes [Moravec 2021]. These influences must be taken into account during construction, production, assembly and operation.

The vibrations that arise in the engagement of the gears manifest themselves by being transmitted to the outer surfaces of the gearboxes and cause increased noise [Bratan 2023]. Internal sources of excitation also affect the noise of gearboxes. Among these sources are the vibrations of bearings and shafts, the vibration transmission system, the construction of the casting or welding of the housing, but also the way the gearbox is stored [Saga 2019, Mascenik 2020, Dziubek 2022]. Another source of irregular vibration excitation is the inaccuracy of the gearing [Juzek 2017, Smeringaiova 2021, Krenicky 2022]. This includes deviations arising from deformations and inaccuracies of other parts of the gearbox, which can affect changes in gear meshing.

Parametrically self-excited vibrations are related to the very principle of engaging the involute flanks of gear teeth [Kuric 2022]. These self-excited vibrations are caused by the change in the stiffness of the teeth during engagement, the change in the number of teeth in the engagement field and the input shock [Stepanov 2014, Kuczaj 2023]. The input shock is determined by the different deformation of the loaded and unloaded tooth at the beginning of the engagement [Krajnak 2021]. This excitation of vibrations occurs even if the meshing teeth are ideally accurate.

The dominant contribution to the noise of gearboxes is almost always the excitation of vibrations in the engagement of the gears. Figure 1 shows an example of the overall evaluation of the noise of the car transmission, where the share of the separated noise of gear engagement (marked as N and marked 3) is in the maximum area of 40% of the noise of the transmission, which contributes 53% to the total noise [Puskar 2024]. The remaining 47% is made up of background noise (marked as Bgr), which can primarily include noise caused by bearings.

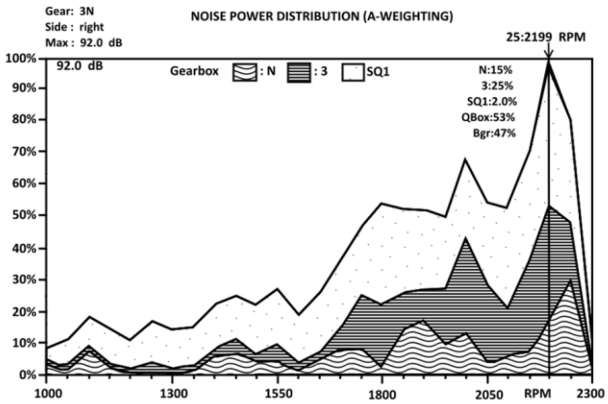

Figure 1. Car transmission noise

One of the factors affecting gear noise is the stiffness of the gearing [Vasko 2020, Kuczaj 2023]. This is influenced not only by the basic parameters of the gearing, such as the number of teeth of the gear wheel, the size of the normalized value of the module, the engagement angle, the angle of inclination of the teeth, but also by the dimensions and shape of the gear body [Wei 2012, Wang 2015, Dziubek 2022].

The contribution is devoted to the issue of the influence of individual factors on the deformation of teeth of gear wheel and thus also the stiffness of the gearing.

2 DEFORMATION OF GEARING TEETH AND MESHING STIFFNESS

Gear teeth are deformed during engagement. Knowledge of the deformation properties of gearing is very important. Due to the complex shape of the teeth, the theoretical determination of the deformation is difficult. Many works are devoted to this issue. Older works were based on the classic theory of elasticity and considered the tooth as a beam that is stressed to bend.

If we consider only the deformation of one tooth (Fig. 2a), then the resulting normal force F deforms the tooth. This is shown by a thin line in the picture. But if we consider the engagement deformation of two co-engaging teeth (Fig. 2b), the resulting deformation in the direction of the normal force is equal to the sum of the deformations of each tooth.

The resulting deformation is denoted by the symbol δi, where i is an index, which if equal to 1 indicates a tooth of the pinion – drive wheel, if equal to 2 indicates a tooth of the driven wheel. This resulting deformation consists of deformation from bending, shearing, deformation at the point of weaving and contact deformation. To simplify the notation, the quantities in the frontal basic direction "t_b" are given below without the index "tb". Then deformation $\delta_{tb} = \delta$, stiffness $c_{tb}=c$ and force $F_{tb}= F$ and width loads $w_{tb}=w$. The other directions are marked with the corresponding indices.

Figure 2b) shows a part of a pair of teeth that touch at point X on the line of contact τ_b in an unloaded state. After loading, the profiles of both engaging teeth are deformed into the shape shown by the dashed line in the corresponding figure. These already deformed tooth profiles cross the line of contact at points X_1 and X_2. The total deformation of this pair of teeth ⍰ can then be determined as the sum of the deformations of both teeth δ_1 and δ_2. Angles $\varphi_{\delta 1}$ and $\varphi_{\delta 2}$ are further shown in the figure. These are the angles through which the individual wheels must turn in order to meet again at point X, as they actually do.

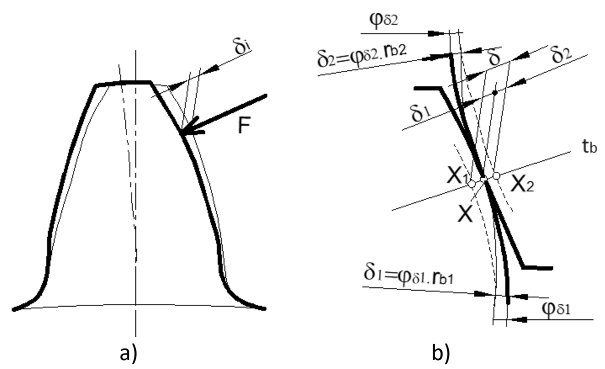

Figure 2. Deformation of a) one tooth, b) one pair of meshing teeth
F - force, X and $X_{1,2}$ - points of contact, δi - tooth deformation, τ_b - line of contact, $\varphi_{\delta 1}$ - rotation angles, $r_{b1,2}$ - base radius

There, along the length of the meshing line, the engagement of two pairs of teeth with one is alternated with the engagement of two spur gears with straight teeth. Therefore, in the engagement of these gears, the involute engagement duration coefficient is greater than 1. The resulting deformation is then equal to the sum of the corresponding components of the tooth deformations, with respect to the engagement line. On the parts marked as AB and DE of the engagement line, the total deformation is equal to the sum of the partial deformations of the teeth of two pairs of teeth engaging together [Kazi 2017, Monkova 2019]. On the segment BD of the engagement line, the total deformation is equal to the sum of the deformations of the teeth of one pair of teeth of spur gears with straight teeth. A typical course of the total deformation of the teeth of spur gears with straight teeth is shown in Fig. 3.

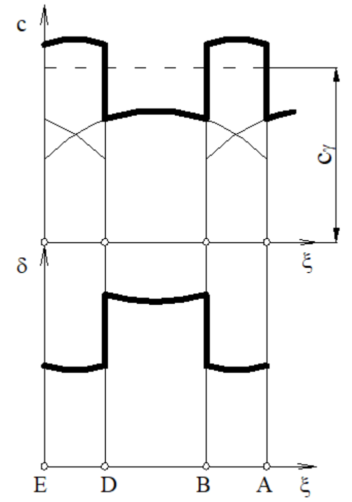

Figure 3. Course of total stiffness and deformation of spur gearing
c - stiffness of gearing, δ -deformation of gearing c_γ - meshing stiffness

The deformation of the gearing is usually expressed quantitatively by the stiffness of the gearing, which is defined as the ratio of the load (longitudinal or width) to the deformation. The stiffness of individual pairs of spur gear teeth changes along the meshing line.

In general, the resulting stiffness c is defined by this formula:

$$c = \frac{w}{\delta} = \sum_p c_p \tag{1}$$

where w - total width load of the gearing [N/mm],

δ - resulting deformation [μm],

p – index, p = I, II.

For total width load of the gearing:

$$w = w_I + w_{II} \tag{2}$$

Two pairs of teeth I and II engage at the same time on the section of two-pair meshing (section AB and DE), which corresponds to the parallel model of two springs. However, when the springs are connected in parallel, the resulting stiffness is equal to the sum of the partial stiffnesses according to the second part of the previous equation. In the case of direct gearing, the resulting stiffness along the meshing line, as well as the corresponding deformation shown in Figure 3.

The resulting gearing stiffness c and deformation δ of the spur gearing changes periodically along the engagement path. This period is equal to the frontal basic pitch of the p_{tb}. In approximate calculations, the average value of the resulting stiffness c_γ is calculated during the entire span (Figure 3). We call this stiffness the meshing stiffness of the gearing and it is constant for the given gearing and given by the relation:

$$c_\gamma = \frac{1}{p_{tb}} \cdot \int_0^{p_{tb}} c(\xi)d\xi = \int_0^1 c(\bar{\xi})d\bar{\xi} \tag{3}$$

Typical value of meshing stiffness $c_\gamma = 20$ N/mm.μm.

A lot of work is devoted to the stiffness of spur gearing [Krish 2011, Magerramova 2022]. Older works were based on the classical theory of elasticity and considered the tooth as a woven beam. The current trend is based on the use of modern stiffness calculation methods, such as the use of the differential method, the finite element method, the boundary element method, etc. [Kagathara 2021].

The stiffness of one pair of spur gear teeth depends on the shape of both teeth. It means that it depends on:

- the number of teeth of the pinion and wheel z_1, z_2,
- the pressure angle α_n,
- the gearing height factor κ,
- the unit displacement of the gears at the corrected gearing x_1, x_2.

As z_1, z_2, α_n and x_1, x_2 increase, the stiffness increases, but as κ increases, it decreases [Pastircak 2017]. Stiffness does not depend on the size of the teeth, i.e. on the modulus m_n.

3 INFLUENCE OF BODY SHAPE ON DEFORMATION AND STIFFNESS OF SPUR GEAR TEETH

The deformation as well as the stiffness of the teeth of spur gears with straight teeth is not constant for all the teeth of the examined gears. Its value depends on the shape of the teeth, i.e. on the basic parameters of the investigated straight spur gear, such as the number of teeth, gear module, engagement angle, gear width, gear correction and modification [Tejani

2017]. Another parameter that affects these parameters is the design solution of the shape of the gear wheel body. We can say that reducing the weight of the gear body influences the deformation and stiffness of the gearing.

Design solutions for the shape of the gear bodies depend on several factors such as the size of the wheel, material, use or production method. Cast, forged or welded gears are used for larger gears.

3.1 *Effect of rim thickness on deformation and stiffness of the gearing*

We will determine the influence of the ring thickness on the deformation and stiffness of the gearing on models of spur gears with straight teeth, the number of teeth is z = 71, the module is m_n = 2.5 mm and the width of the gearing is b = 50 mm. Several models of spur gears were created where the thickness of the web was the same for all models (value f = 15 mm) and the thickness of the rim e was varied (see Fig. 4). The size of the width load w = 100 N/mm was considered, expressed on the basis of a single force F = 5000N.

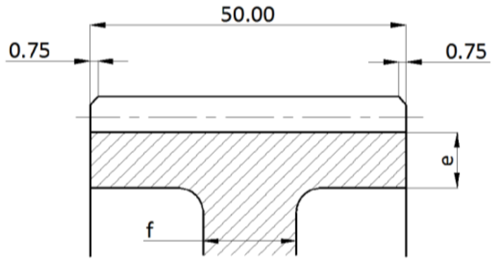

Figure 4. Defining parameters as e - rim thickness, f - thickness of the web

Figure 5 shows the dependence of gear deformation on rim thickness. As can be seen from the obtained results, the deformation of the teeth increases by reducing the thickness of the crown. As can be seen from the obtained values, the deformation changes with a larger change up to the size of the crown equal to 3.5m_n.

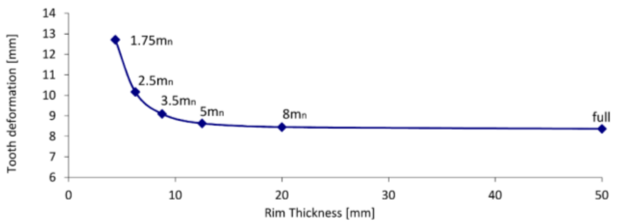

Figure 5. Influence of rim thickness on gearing deformation

Figure 6 shows the effect of rim width on gear stiffness. As the thickness of the rim increases, the stiffness of the teeth also increases. A rim thickness smaller than a value equal to 3.5 mm affects the stiffness of the gear more significantly, therefore it is advisable to choose a rim thickness equal to or greater than this value.

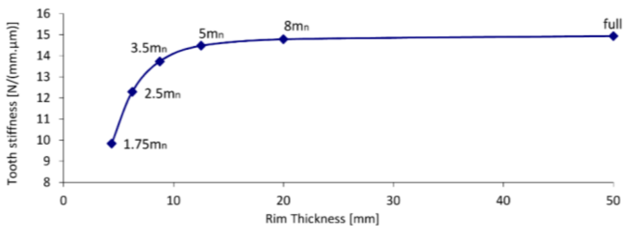

Figure 6. Influence of rim thickness on gear stiffness of teeth

3.2 *The influence of the thickness of the web on the deformation and stiffness of the gearing*

The web thickness f also affects the deformation and stiffness of the gearing (Figure 4). A gear body model was investigated, the web of which was always located in the middle of the width of the gear teeth and whose thickness varied from 10 mm to 50 mm. With web thickness of 50 mm, it will be a full gear wheel without web. Based on the deformation results for individual cases (Figs. 7 and 8), the tooth stiffnesses were determined.

Table 1 shows the results of the influence of the web thickness on the deformation and stiffness of the toothing, determined at the center of the toothing width, if the tooth is loaded at the engagement points A and D. As the results show, when increasing the width of the web f, which is located at the center of the width of the gear wheel the deformation slightly decreases, and the stiffness slightly increases. However, by increasing the width of the web, the course of deformation, as well as stiffness, flattens along the width of the gear wheel.

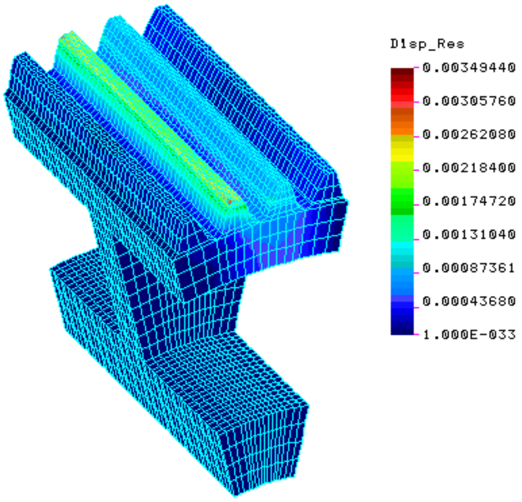

Figure 7. Gearing deformation solution if the number of teeth z=61, the module is m_n=2.5mm, the width of the wheel b=50mm and the web thickness f=10mm

Table 1. Effect of the web thickness on deformation and stiffness of toothing

Engagement point A			Engagement point D		
f [mm]	δ [μm]	c [N/mm.μm]	f [mm]	δ [μm]	c [N/mm.μm]
10	2.749	14.550	10	0.824	48.490
20	2.735	14.625	20	0.819	48.792
30	2,719	14.706	30	0.815	49.062
40	2.709	14.763	40	0.812	49.243
50	2.700	14.814	50	0.809	49.407

The teeth stiffness is not constant even over the face width of the toothing. If the line of contact ends at the edge of the teeth (Figure 8, the stiffness of the teeth is smaller at these edges due to the free end of the tooth lacking a supporting effect.

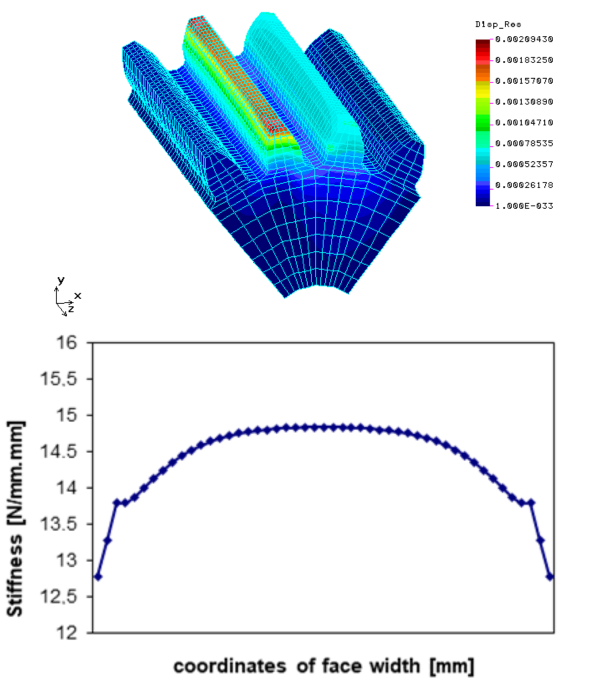

Figure 8. The course of gear wheel tooth stiffness along the width of the load

The web locally affects the change in the stiffness of the toothing along the width of the load, as shown in Figure 9. At the location of the web, there is a local small increase in the stiffness of the toothing.

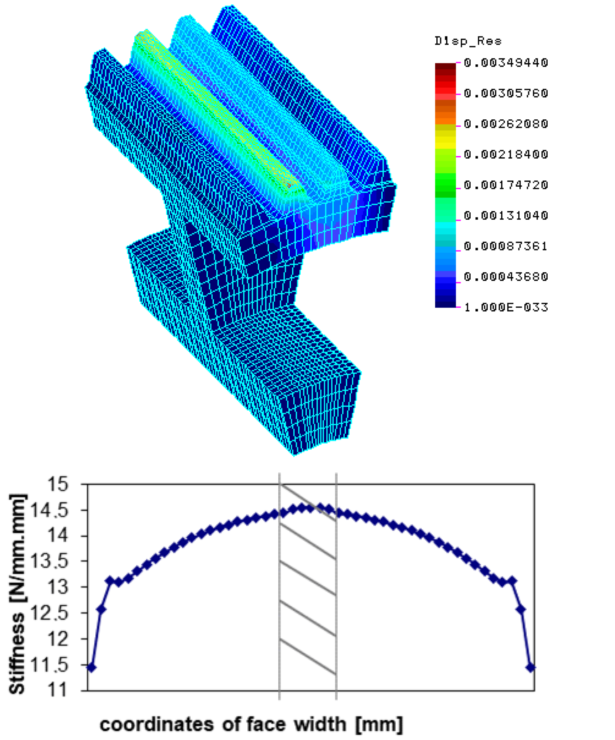

Figure 9. The influence of the location of the web on the course of the stiffness of the gear teeth across the width of the load

However, if the line of contact do not ends in the edge of the face width (Figure 10), there will be a fairly sharp increase in teeth stiffness at the edges of the contact due to the support effect.

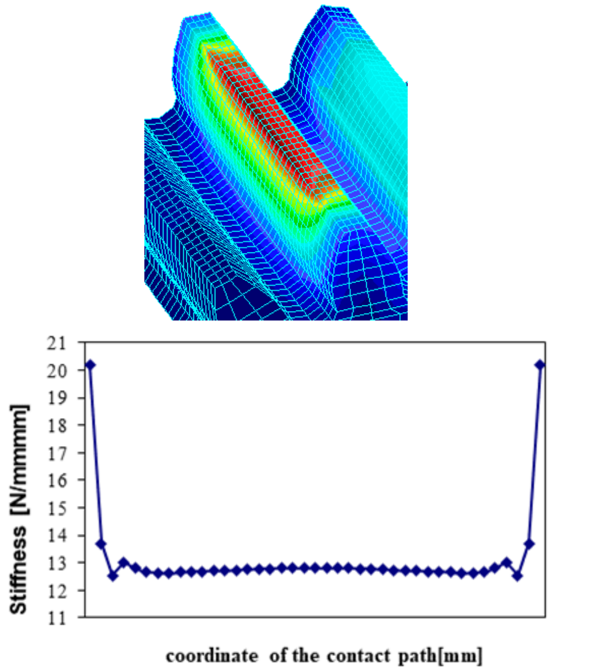

Figure 10. Influence stiffness of teeth to course along the face width of the toothing

Another influence on the deformation and stiffness of the gearing is the location of the web. This study is solved on models of spur gears with a web width of f=10mm, the position of which changed with respect to the gear width. Figure 11 shows the result of the deformation investigation if the web is placed on the edge of the wheel.

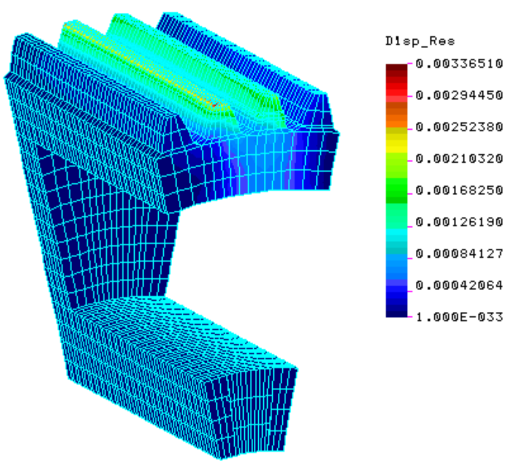

Figure 11. Deformation solved by FEM if the web is located on the edge of the wheel

Figure 12 shows the solution when the web is placed at a distance of 10 mm from the edge.

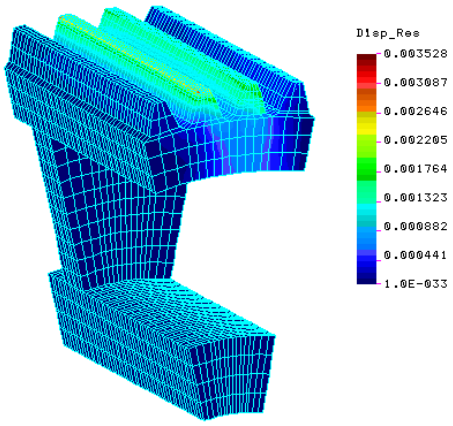

Figure 12. Deformation solved by FEM when placing the web at a distance of 10 mm from the edge of the gear

A model of a spur gear was investigated, which has two webs located along the edges of the width of the gear wheel (Fig. 13).

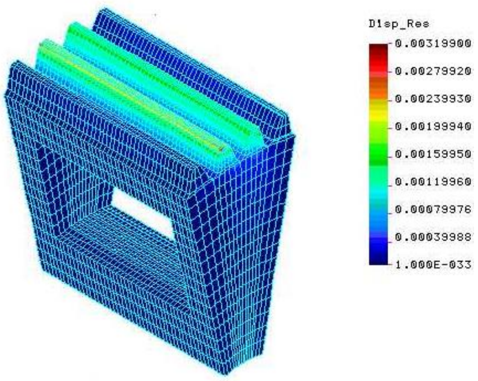

Figure 13. Deformation of gearing solved by FEM when placing I stand at the edges

The minimum value of tooth deformation is obtained if the gear wheel has two webs located along the edges of the toothing (Fig. 13) due to the supporting effect of the webs. The location of the web generally affects the course of deformation as well as stiffness across the width of the spur gear.

Another possible requirement placed on gear shape solutions is the effort to reduce production costs. Such an effort can be achieved by changing the dimensions of the given wheel, the production machines used and, last but not least, the number of produced pieces of the given gears. It goes without saying that the effort to maintain the required stiffness of the gearing is therefore acquired, the knowledge obtained of the minimum permitted dimensions of various parts of the gear wheel is able to speed up the process of solving the given design proposals.

4 CONCLUSIONS

There are many influences that cause noise and vibration in gearboxes. These influences must be taken into account already during construction, production, assembly and operation. Detailed analyses of gear box manufacturers have shown that by improving the accuracy of the gear wheels, the noise level of the gear unit will be reduced only to a very small extent. Changes in the shape of the tooth, its geometry and changes in production technology can achieve a more significant reduction in noise in the transmission mechanism. A significant source of noise and vibration in the gearbox is the change in the stiffness of the gearing during the contact of the gears. Based on the analysis of the influence of individual parameters on the stiffness of the gearing, it can be concluded that the course of the overall stiffness of the teeth of the gearing increases with the increasing number of teeth. It is a result of the shape of the lateral curve of the tooth. The higher the number of teeth, the greater the radius of curvature of the side of the tooth, and thus the bending stress on the tooth is smaller.

The design of the shape of the body of the gear wheels has an effect on the change in the course of the stiffness of the gearing. As the thickness of the body rim increases, the value of the gear stiffness increases. Here, the value of the thickness of the crown equal to 3.5 times the module is important. When choosing a rim thickness greater than this value, the value of the stiffness of the toothing changes to a lesser extent. As the thickness of the web of the wheel body increases, the stiffness of the gearing increases. The stiffness is not constant even across the width of the toothing. It is more advantageous if the line of contact - contact ends in the middle is smaller than the width of the teeth. Otherwise, there is an increase in the stiffness of the toothing at the end points of contact on the contact line.

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