VISUALIZATION OF VECTORS AND THEIR APPLICATION IN MECHANICAL ENGINEERING

MAJA CULETIC CONDRIC[1],

MATILDA SIROVINA[1], VEDRAN NOVOSELAC[1], MARIJA STOIC[1]

[1]University of Slavonski Brod, TECHDpt, Slavonski Brod, Croatia

DOI: 10.17973/MMSJ.2025_06_2025046

e-mail to corresponding author: mccondric@unisb.hr

In mathematics education, geometry presents students with greater difficulties than other mathematical content, and one of the main reasons for this is the development and ability of visualization. Visualization refers to the ability to present ideas, information and concepts through graphical or pictorial representations. Many empirical studies as well as theoretical statements, for example van Hiel's theory, indicate that visualization or recognition is the first step in geometric thinking and reasoning, followed by analysis, informal deduction, deduction and rigor. Vectors as a concept and as a unit of instruction are learned in elementary school and appear in high school through to college classes, where the culmination of their application is reached in the analysis of stresses, strains, vibrations, fluid dynamics and many other segments of mechanical engineering.

In the theoretical part of the paper, the mathematical and graphical representation of vectors is combined with the processing of arithmetic operations with vectors. How and in what way to explain and visualize certain vector quantities or formulas for scalar and vector products to students is a major challenge for professors. The experimental part of the paper shows some concrete examples of the application of vectors that students learn in mathematics but also in other professional courses at the Technical Department. The results of a survey of the first year of professional undergraduate studies at the Technical Department will also be presented, providing information about students attitudes and their approach to visualization and teaching content.

KEYWORDS

mechanical engineering, vectors, visualization, van Hiel's theory

1 INTRODUCTION

Mechanical engineering, as one of the most important technical fields, uses various mathematical concepts and tools to solve complex problems. One of these important tools are vectors, which enable the analysis and modeling of geometric shapes and the solving of systems of equations. Vectors in mechanical engineering have a wide range of applications, including the design and analysis of mechanisms, the calculation of static and dynamic loads, design simulation, the design and control of advanced robotic systems, the calculation of the thermal conductivity of materials and many others. It is important to know that vectors and vector spaces are the first step, i.e. a crucial part of calculations in mechanical engineering.

In the late 1950s, more precisely in 1959, the Dutch mathematicians, a married couple, Dieke van Hiele-Geldof and Pierre van Hiele, wrote their doctoral theses at the College of Utrecht in the Netherlands and published the well-known Van Hiele theory of geometric reasoning [Van Hiele 1999]. They presented their theory of geometric thinking based on their own teaching experience and research. Pierre van Hiele wanted to find the reason for students' poor performance in learning geometric content, while Dieke van Hiele-Geldof aimed to find specific teaching methods that would enable students to achieve better success in learning and understanding geometry. They explained their theory in terms of stages or phases of thinking that one goes through when learning geometry [Howse 2015]. Based on their theory, detailed research and studies, they revised the geometry curriculum, which led to improvements in the learning and teaching of geometry in the United States at the end of the 20th century [Musser 2007].

Thus, there are five stages of thinking that a student goes through on the way to acquiring the ability to do formal proofs and understand non-Euclidean geometry. At each successive level, new knowledge and skills are acquired, and in order to progress to a higher level, the previous level must be mastered. The age and years of an individual have no influence on the transition from one level to the next, but the understanding and mastery of certain materials and geometric concepts do [Vlasnović 2014].

Visualization can be defined in two ways: first, as the creation of mental images and second, as the visual representation of information. Visualization is a powerful tool because it allows the brain to process information faster and more efficiently than is possible with text alone. We are visual beings - we take in most information from our environment through sight. Visualization allows us to see patterns, trends and relationships that we might not notice by reading text alone.

The aim of this paper is to combine the pedagogical and didactic aspects, i.e. the visualization of a certain concept such as vectors and different applications of vectors in mechanical engineering, and to emphasize the importance of the visual representation of vectors in solving complex problems. Understanding vector operations and other mathematical concepts provides mechanical engineers with a powerful tool to help them develop innovative projects and solve challenging problems in their field.

2 VISUALIZATION AS A TEACHING METHOD

2.1 Visualization

Visualization is the presentation of information or data in a visual format, e.g. in the form of diagrams, maps, charts, images or animations. The aim of visualization is to facilitate the understanding of complex concepts, to show similarities, trends or correlations and to enable a faster and more intuitive interpretation of the data. In addition, one of the main goals of visualization is to convey complex data in a way that is easy to understand and intuitive for the viewer. In this way, visualization can help uncover patterns, trends or anomalies in data, which can be useful in various fields such as science, business, medicine, education and social sciences.

Visualization plays a key role in learning geometry by giving students a more concrete insight into abstract concepts, enhancing geometric intuition, facilitating understanding of theorems and proofs, encouraging experimentation and exploration, providing different learning modalities, and stimulating creativity. The use of visualization in college-level geometry instruction provides students with a deeper understanding of complex mathematical concepts, enhances their critical thinking and problem-solving skills, and prepares them for further research and professional practice in mathematics and related disciplines.

2.2 Spatial dawn

The German mathematician Hans Freudenthal said: "Geometry is a tangible space; it is the space in which a child breathes, lives and moves. It is the space that a student must get to know, explore and conquer in order to be able to live, breathe and move better in it." In line with these words, we observe significant problems in navigating the surrounding space. Students have great difficulty connecting the two- and three-dimensional worlds, concepts, shapes and figures and understanding geometry as such. Geometry as a branch of mathematics is more interesting to students than algebra, but it is the visualization of space that poses challenges and misunderstandings for both students and teachers, and yet this geometry is essential to a person's daily life. Numerous professions such as civil engineering, mechanical engineering, architecture, geography, shipbuilding, various design fields and the like cannot function without the visualization of space. An important concept is spatial visualization, which is defined as the ability to visualize space, i.e. to remember the shapes and sizes of geometric figures and to recognize all their mutual positions and relationships [Radović 2012]. In the area of shape and space of the National Curriculum, the development of spatial visualization is one of the very important tasks in mathematics education. In today's world, where IT technology is very much to the fore, the question arises as to the nature and extent of the use of computer programs in mathematics lessons, especially in lessons dealing with geometric content.

2.3 Van Hiele's theory

The Dutch husband and wife, mathematics teachers, Pierre and Dieke describe the theory in terms of five levels of geometric thinking. The most important result of van Hiele's theory is the identification of all five levels, starting from the zero level up to the fourth level, each of which we have described. They present their theory of geometric thinking based on their own teaching experience and research. Pierra van Hiele tries to find the reason for students' lack of success in learning geometric content, and Dieke van Hiele-Geldof tries to discover specific teaching methods that enable students to have more success in learning and understanding geometry. They explain their theory in terms of the level or levels of thinking one goes through when learning geometry. In some research [Baranović 2015], the numbering of the levels is used from one to five so that there is no confusion, for example, if someone is at level two, it is actually the third level of geometric thinking.

Many have written about this theory and it has been confirmed using various models, and there is no longer any doubt about its validity, so every person, whether student or student, is certainly at some level outside of Hiele's theory of geometric thinking. Students in the lower grades of elementary school are usually at level zero, students in the upper grades of elementary school are at level one, while it is rare for an eighth grader to be at level two. High school students should generally be at level three, but most also show level two, and engineering and math students would show the highest level four.

The stages of van Hiele's theory of geometric thinking are thus:

a) Level 0 – Visualization or recognition
b) Level 1 – Analysis
c) Level 2 – Informal deduction
d) Level 3 – Deduction
e) Level 4 – Rigor

In order to successfully master the aforementioned stages of van Hiele's theory of geometric thinking, he also specified five stages for the successful process of learning and mastering the learned stages in his theory of Pierre van Hiele. These learning stages are supported by examples and activities that the teacher and student carry out in order to reach a certain level. The learning phases are divided into:

1. Phase of questions and information (Inquiry/Information)
2. Phase of targeted orientation (Directed orientation)
3. Explanation phase (Explicitation)
4. Open activity phase (Free orientation)
5. Integration phase (Integration)

For the purposes of this article, only level 0 is explained. At this lowest level, thinking is holistic, i.e. a person recognizes geometric objects visually based on their overall appearance, without paying attention to their components and without identifying their properties at the same time. Students at this level perceive space as something that surrounds them, and when they judge geometric shapes by their appearance, they generally see them as a whole. They learn the names of figures and geometric concepts on the basis of their shape, not on the basis of their properties (sides, angles, etc.).

A person who has mastered this level recognizes a triangle or any geometric figure, a square or a rectangle rotated in a plane, i.e. they recognize it in any of its positions (Figure 1 b) in relation to the standard position when one side is parallel to the bottom edge of the paper. People who have not yet reached this stage of visualization do not notice the roundness of the sides, but consider the entire figure to be triangular and claim that they are triangles because they are in the standard position they have learned to recognize by then (Figure 1 a). After the recognition process, people begin to categorize figures or bodies into groups, but again only according to their appearance as a whole because they look similar.

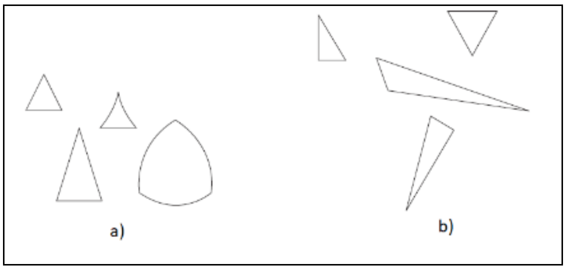

Figure 1. Group of figures perceived as triangles (a) and group of figures not perceived as triangles (b) [Musser 2007]

3 VECTORS AND CALCULATION OPERATIONS

3.1 Vectors in the course of history

Although vectors are a fundamental concept in mathematics today, their origins lie in physics. They were first used by Simon Stevin in 1585, and a hundred years later Isaac Newton noted that acceleration is always in the direction of the force that causes it. In the 16th and 17th centuries, scientists discovered many properties of vectors, but did not use the term vector itself. The word vector comes from Latin and means to carry or transmit (veho, vectus). In 1637, René Descartes introduced the coordinate system, which enabled significant advances in the methods of calculation with vectors. Modern vector theory began to develop in the middle of the 19th century, particularly in connection with the geometric representation of complex numbers. In the 19th and 20th centuries, vector theory evolved from a tool for mechanics and geometry to a modern and powerful language of applied mathematics. Today, vectors are used in various fields of science, from technical to economic disciplines.

3.2 *Basic concepts of vectors*

A vector is a directed line where one edge point is defined as the start (handle) and the other as the end (tail). We denote the vector whose start point is A and end point is B as $\overrightarrow{AB}$. The end point of the vector is always marked with an arrow. Vectors are also designated with the lowercase letters $\vec{a}$, $\vec{b}$, $\vec{c}$, etc. They are sketched as shown in Figure 2. Each vector is determined by its length, direction and orientation [Žarinac-Frančula 1990].

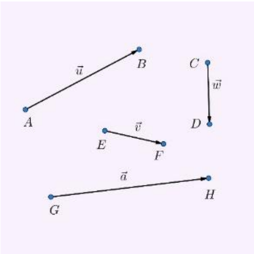

Figure 2. Vectors

Vectors are often represented by coordinates that describe their end points with respect to the starting point (usually the origin of the coordinate system). It is well known that points are part of both the plane and space. If there is a fixed point O in a space E, then a vector $\overrightarrow{OT}$ can be assigned to each point T in the space (Figure 3). Such a vector is called the position vector of point T ($\vec{r_T} = \overrightarrow{OT}$) [Pačar 2013].

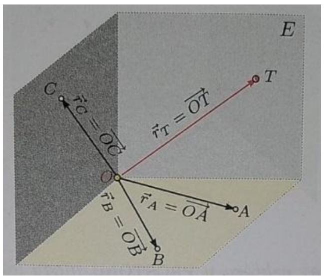

Figure 3. The vector of the position of the point T in relation to the origin O [Pačar 2013]

The vectors $\vec{r_A}$, $\vec{r_B}$, $\vec{r_C}$ can also be written as $\vec{a}$, $\vec{b}$, $\vec{c}$. The ordered triple ($\vec{a}$, $\vec{b}$, $\vec{c}$) is called the basis of the vector space, and each vector $\vec{x}$ that belongs to this space can be written as a linear combination of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ (Figure 4).

$$\vec{x} = x_1\vec{a} + x_2\vec{b} + x_3\vec{c} \quad (1)$$

where: x_1, x_2, x_3– real numbers (scalars).

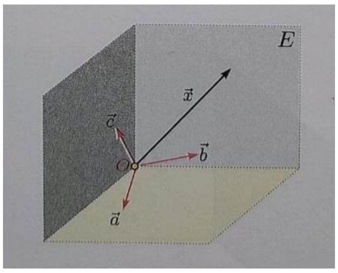

Figure 4. *Linear combination of vectors $\vec{x}$ in space [Pačar 2013]*

3.3 *Arithmetic operations with vectors*

Arithmetic operations with vectors allow the manipulation of vector quantities in different ways, which is crucial for their application in various scientific and technical disciplines. Below you will find the basic arithmetic operations with vectors: Addition, subtraction, scalar multiplication, dot product and cross product.

Vector addition is an operation in which two vectors are combined to form a new vector. There are two approaches to vector addition: analytical and graphical.

The analytical approach is defined as a function that assigns a new vector $\vec{c} = \vec{a} + \vec{b}$ to the pair of vectors $\vec{a}$ and $\vec{b}$. The sum of two vectors $\vec{a}(x_a\vec{i}, y_a\vec{j}, z_a\vec{k})$ and $\vec{b}(x_b\vec{i}, y_b\vec{j}, z_b\vec{k})$ is given by:

$$\vec{a} + \vec{b} = (x_a + x_b)\vec{i} + (y_a + y_b)\vec{j} + (z_a + z_b)\vec{k} \quad (2)$$

where are:

$\vec{a}$, $\vec{b}$ - vectors a and b

x_a, y_a, z_a- coordinates of vector a (scalars)

x_b, y_b, z_b - coordinates of vector b (scalars)

$\vec{i}, \vec{j}, \vec{k}$ - rectangular or Cartesian coordinates of the vector.

The graphical addition of vectors is most frequently represented by the parallelogram rule or the triangle rule. According to the parallelogram rule, two vectors $\vec{a}$ and $\vec{b}$ are placed so that they have a common origin. A parallelogram is then constructed in which these vectors are adjacent sides, as shown in Figure 5. The diagonal of the parallelogram, which starts from the common origin, represents its sum $\vec{c} = \vec{a} + \vec{b}$ [Pačar 2013].

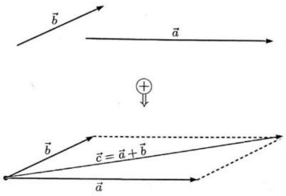

Figure 5. Illustration of the addition of vectors according to the parallelogram rule [Pačar 2013]

According to the triangle rule, two vectors $\vec{a}$ and $\vec{b}$ are placed so that the end of vector $\vec{a}$ is the handle of vector $\vec{b}$, and then the beginning $\vec{a}$ and the end $\vec{b}$ are connected by a new vector $\vec{c}$. The new vector in Figure 6, which has a start at the beginning of vector $\vec{a}$ and an end at the end of vector $\vec{b}$, represents their sum $\vec{c} = \vec{a} + \vec{b}$.

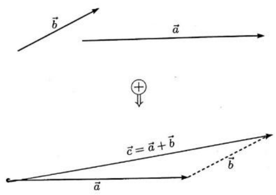

Figure 6. Illustration of the addition of vectors according to the triangle rule [Pačar 2013]

The addition of vectors has several important properties, such as: closedness (the result of adding two vectors is a new vector), commutativity, associativity, the zero vector $\vec{0}$ and the opposite vector $(-\vec{a})$.

Vector subtraction can be interpreted as the addition of a vector to its opposite vector. If $\vec{a}$ and $\vec{b}$ are two vectors, then the subtraction of $\vec{a} - \vec{b}$ is equal to the addition of $\vec{a}$ and $\vec{b}$. Vector subtraction also has its properties, which are almost the same as the properties of addition with one difference, i.e. anticommutativity applies to vector subtraction.

Multiplying a vector by a scalar results in a vector whose length changes by a scalar factor but whose direction remains the same (or is reversed if the scalar is negative). If $\vec{a}$ is a vector and λ is a scalar, then the multiplication is defined as $\vec{b} = \lambda \cdot \vec{a}$. The multiplication of a vector by a scalar is an operation that can be represented analytically. If we have a vector $\vec{a}(x_a\vec{i}, y_a\vec{j}, z_a\vec{k})$and a scalar ($\lambda$), their product is

$$\lambda \cdot \vec{a} = \left((\lambda \cdot x_a)\vec{i}, (\lambda \cdot y_a)\vec{j}, (\lambda \cdot z_a)\vec{k}\right) \tag{3}$$

where: x_a, y_a, z_a – real numbers (scalars)

The multiplication of a vector by a scalar, as well as the addition and subtraction of vectors, has properties such as distributivity (in relation to the vector factor), distributivity (in relation to the scalar factor), quasi-associativity.

The scalar product of a vector (inner or in-product) is a function that adds a scalar (real number) to a pair of vectors $\vec{a}$ and $\vec{b}$. The scalar product of two vectors $\vec{a}(x_a\vec{i}, y_a\vec{j}, z_a\vec{k})$ and $\vec{b}(x_b\vec{i}, y_b\vec{j}, z_b\vec{k})$ is defined as:

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi \tag{4}$$

$$\vec{a} \cdot \vec{b} = x_a \cdot x_b + y_a \cdot y_b + z_a \cdot z_b \tag{5}$$

where: ϕ is the angle between the vectors $\vec{a}$ and $\vec{b}$. The scalar product of a vector results in a scalar value, not a vector. It is used to calculate the angle between vectors, to check the orthogonality of vectors and to calculate the projection of one vector onto another.

The graphic representation of the scalar product is shown in Figure 7.

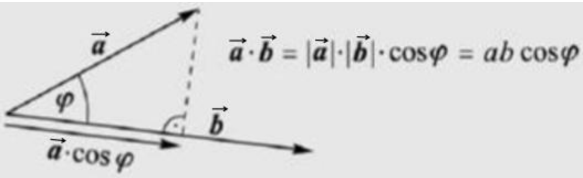

Figure 7. Scalar product of vectors [Pačar 2013]

The vector product of vectors (outer or ex-product) is a function that joins a pair of vectors $\vec{a}$ and $\vec{b}$ with a vector $\vec{c} = \vec{a} \times \vec{b}$ which is perpendicular to both the vector $\vec{a}$ and the vector $\vec{b}$. It is defined as:

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi \tag{6}$$

where φ is angle between vectors $\vec{a}$ and $\vec{b}$. The component solution of the vector product of vectors $\vec{a}(x_a\vec{i}, y_a\vec{j}, z_a\vec{k})$ and $\vec{b}(x_b\vec{i}, y_b\vec{j}, z_b\vec{k})$ is:

$$|\vec{a} \times \vec{b}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_a & y_a & z_a \\ x_b & y_b & z_b \end{vmatrix} \tag{7}$$

$$= (y_a z_b - z_a y_b)\vec{i} + (z_a x_b - x_a z_b)\vec{j} + (x_a y_b - y_a x_b)\vec{k}$$

The orientation of the vector $\vec{c} = \vec{a} \times \vec{b}$ is determined by the right-hand rule. If the fingers show the direction from the vector $\vec{a}$ to the vector $\vec{b}$, the thumb then shows the orientation of the vector $\vec{c}$. The vector product of vectors also has its own properties, such as: anticommutativity, distributivity and homogeneity [Pačar 2013].

The graphic representation of the vector product is shown in Figure 8.

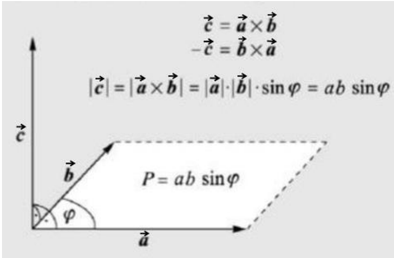

Figure 8. Vector product of vectors [Pačar 2013]

4 EXPERIMENTAL PART

In the experimental part of the work, several concrete examples of the application of vectors that students learn in the professional courses of the undergraduate study of Production Engineering will be shown. In addition, the results of a survey conducted among the students are presented, in which the students' attitudes and attitudes towards visualization, teaching content and the literature read for a particular course become visible.

4.1 Application of vectors in mechanical engineering

Vectors are an indispensable tool in mechanical engineering because they enable precise modeling and analysis of forces, moments, velocities and other physical quantities that are crucial for the design and analysis of machines, structures and mechanisms.

A vector representation of forces allows engineers to accurately model the effect of forces on a structure or mechanism. Each force acts in a specific direction and has its own magnitude (intensity), which can be described mathematically by a vector. When analyzing the load on a structure, for example, force vectors are used to determine the balance and distribution of forces within the system [Sirovina 2024].

Example 1: Vectors in statics - forces

The maximum circumferential force on the drive rope (rope pulley with guide groove) of the elevator is to be calculated. Before you calculate the elevator yourself, you need to know the requirements that the elevator must meet. The following values were selected for this calculation: Drive type: electric; Load capacity: 750 kg (10 persons); Lifting height: 15 m; Number of cells: 6; Travel speed: 1.0 m/s; Car dimensions (HxWxL): 2200 x 1400 x 1300 mm. [Matić 2023]

The maximum circumferential force on the rope is (Figure 9): $F_o = F_1 - F_2 = G_t + G_k + G_s - G_u$

where is:

F_o - maximum circumferential force on the rope,

F_1 - force on the rope which amounts to $F_1 = G_t + G_k + G_s$,

F_2 - force on the string which amounts to $F_2 = G_u$,

G_t - load weight (in the picture Q),

G_k - cabin weight,

G_s - weight of load-bearing ropes,

G_u - counterweight weight.

Applying the expression for the weight of the counterweight $G_u = G_k + \frac{G_t}{2}$ gives: $F_o = \frac{G_t}{2} + G_S$. Before listing, you should calculate: $G_t = m_t \cdot g = 750 \cdot 9.81 = 7357.5$ N,

where: m_t - load mass, g - gravitational acceleration I

$G_s = z \cdot m_s \cdot (H + 3) \cdot g = 4 \cdot 0.5 \cdot (15 + 3) \cdot 9.81 \approx 353$ N, where: $z = 4$ – selected number of ropes, $H = 15m$ - lifting height, m_s = 0.5 kg/m - mass of rope per meter of length (previously calculated).

After entering the calculated values, the result is:

$$F_o = \frac{G_t}{2} + G_S = \frac{7357.5}{2} + 353 = 4031.75\ N$$

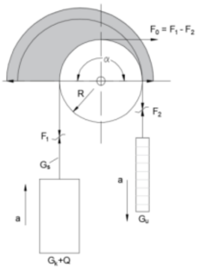

Figure 9. Display of forces on the drive shaft [Matić 2023]

The moment of force can also be represented by a vector. It represents the rotation around a certain axis and its vector determines the direction and magnitude of the moment of force acting on the rotating system. This concept is of crucial importance when analyzing rotating mechanisms such as motors, power transmissions and the like.

Example 2: Moment of force - vectors

A force of 20 N acts on one end of the lever at a distance of 0.5 m from the support as shown in Figure 10. How much force must act on the other end of the lever at a distance of 0.2 m from the support for the system to be in equilibrium? [Sirovina 2024]

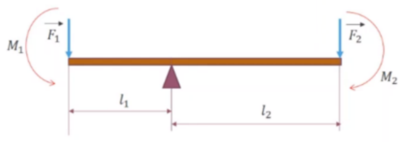

Figure 10. Lever on support with drawn forces and distances [Sirovina 2024]

For the system to be in equilibrium, the moments at the ends of the lever must be equal, so we say:

$M_1 = M_2$.

Since it is known that the moment is the effect of the force on the arm, $M = F \cdot l$ follows: $M_1 = F_1 \cdot l_1$ and $M_2 = F_2 \cdot l_2$

After inserting and rearranging the equation, the following result is obtained:

$F_1 \cdot l_1 = F_2 \cdot l_2$

$$F_2 = \frac{F_1 \cdot l_1}{l_2} = \frac{20 \cdot 0.5}{0.2} = 50\ \text{N}.$$

where:

F_1 - force acting on the first end of the lever,

l_1 - length of the first arm of the lever,

l_2 - length of the second arm of the lever,

F_2 - force acting on the rear end of the lever.

In kinematics, vectors are used to describe the velocity and acceleration of bodies or particles. Velocity is a vector that determines the direction and speed of motion, while acceleration is the change in velocity over time. The use of vectors in kinematics enables the precise modeling and analysis of complex movements, such as translation, rotation and their combinations.

Example 3: Vectors in kinematics

Two cars are moving uniformly in the same direction at speeds of 80 km/h and 50 km/h as shown in Figure 11. Both cars start moving at the same time and the distance between the first and second car is 2 km. Determine the time t at which the first car will catch up with the second [Sirovina 2024].

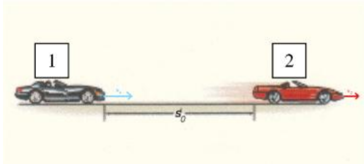

Figure 11. Movement of one car in front of another [Sirovina 2024]

Since the cars move in uniform rectilinear motion, the expression for the path in uniform rectilinear motion $s = v \cdot t$ is used. The following applies to the first car $s_1 = v_1 \cdot t$.

where:

s_1 - total distance of the first carriage,

v_1- speed of the first carriage,

t - duration of the movement at the time at which $s_1 = s_2$.

As the second car is in front of the first, the distance between the two cars s_0 is added to the expression so that the following applies to the second car $s_2 = s_0 + v_2 \cdot t$.

where:

s_2 - total distance of the second car, s_0 - distance of the first car from the second, v_2 - speed of the second car.

At the moment when the first car catches up with the second, the distances of the two cars are equal. Therefore it's worth $s_1 = s_2$.

After inserting and rearranging the equation, you get the following result: $v_1 \cdot t = s_0 + v_2 \cdot t$ and $(v_1 - v_2) \cdot t = s_0$

$$t = \frac{s_0}{v_1 - v_2} = \frac{2}{80 - 50} = \frac{1}{15}\ \text{h} = \frac{1}{15} \cdot 3600 = 240\ s$$

In the analysis of material mechanics, vectors are used to describe deformations and stresses in materials or structures. Stresses are represented as tensor vectors that describe internal forces within the material, while deformations determine the changes in the shape of the material under the influence of external forces.

Example 4: Stress and strain vectors

Given is the stress state at a body point in the rectangular $(0zyx)$ coordinate system, Figure 12, which forms the stress tensor matrix σ_{ij} [Vnučec 2008]:

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} = \begin{bmatrix} 100 & 30 & 0 \\ 30 & 180 & 0 \\ 0 & 0 & -100 \end{bmatrix} \text{MPa}$$

It is necessary to calculate the principal stresses σ_1, σ_2, σ_3 with an oriented element at a body point.

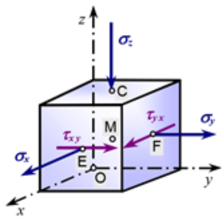

Figure 12. State of stress in a point of the body [Vnučec 2008]

It can be immediately concluded that the normal stress in the z-axis direction will also be the main stress in the 3-axis direction $\sigma_3 = \sigma_z = -100$ MPa.

The stress σ_z does not affect the element in the (xy)plane, and the remaining two main stresses σ_1 and σ_2 can be determined analytically:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{100 + 180}{2} \pm \sqrt{\left(\frac{100 - 180}{2}\right)^2 + 30^2} = 140 \pm \sqrt{40^2 + 30^2}$$

$$= 140 \pm 50 \text{ MPa}.$$

After listing and arranging the expressions, two possible solutions are obtained:

$\sigma_1 = 140 + 50 = 190$ MPa, $\sigma_2 = 140 - 50 = 90$ MPa.

The main stress directions 1 and 2 are determined from the expression: $\tan 2\varphi_0 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{30}{-40} = -0.75$,

and follows $2\varphi_0 = -36.87°$, or $\varphi_0 = -18.435°$.

The main direction 1 is deviated from the y-axis in the clockwise direction by an angle φ_0, and the direction 2 is also deviated from the x-axis by the same angle, when rotating the element around the z-axis in the (x, y) plane. The oriented element with principal stresses is shown in Figure 13.

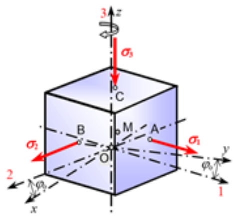

Figure 13. Oriented element with principal stresses [Vnučec 2008]

4.2 Discussion of practical examples

Each of the examples demonstrates the specific role of vectors in various aspects of mechanical engineering, from basic statics to complex stress analysis. The strengths of individual examples lie in their clarity and technical precision, while their weaknesses are most often related to idealizations and limited scope. These examples provide a basic foundation for understanding the concept of vectors in various branches of mechanical engineering, but they lack a connection to modern tools and methods such as numerical simulation or CAD models that are standard in engineering practice today. To the extent that an individual lecturer manages to mention all of the above in his lecture, the student is still expected to deepen his knowledge by using a variety of available literature. For a comprehensive approach to engineering problems, it is important to build on these basic models by including realistic conditions, using computer simulations, and connecting theory with practical applications in industry.

4.3 Survey research

Students of the first year of the professional undergraduate study in Production Engineering and students of the professional undergraduate study Informatics and Information Technologies of the Technical Department participated in the experimental study. A total of 44 people participated, including 20 full-time and 24 part-time students. The survey was conducted using a short questionnaire. The survey was anonymous and voluntary. It took about 5 minutes to complete and consisted of two parts with a total of 9 questions. The collected data was processed in Excel and the Statistica program.

The first part contained four questions about the general and socio-demographic characteristics of the students and the type of high school attended, Table 1. The vast majority of students came from vocational-technical schools, namely 31 students or 70%, followed by 8 students or 18% who attended high schools, while the rest attended some social science programs. When asked "How regularly did you attend lectures and tutorials for all courses?", the responses showed that full-time students attended more than 80% of the courses and part-time students attended more than 50%, with only one person indicating that they did not attend classes regularly.

Question/Answer	Number of respondents
1. Gender	
Male	34
Female	9
Refused to say	1
2. Completed secondary school:	
Gymnasium	8
Four-year vocational school - technical majors	31
Four-year vocational school - social majors	2
Three-year vocational school (passed 4th grade)	3
Five-year secondary school - medical majors	0
3. Study program:	
Professional undergraduate study Production Engineering - full-time	20
Professional undergraduate study Production Engineering - part-time	5
Professional undergraduate study of Informatics and Information Technologies - part-time	19
4. I attended lectures and exercises of all courses:	
regularly - more than 80% (full-time study)	24
regularly - more than 50% (part-time study)	19
not regularly	1

Table 1. Data for the first part of the survey questionnaire.

The second part contained five questions in which students were asked to express their attitude or level of agreement with statements from a student's perspective about geometric content and the importance of visualization in the classroom. The fifth question was a Likert scale ranging from -2 to 2, with -2 indicating strongly disagree, -1 indicating partially disagree, 1 indicating partially agree and 2 indicating strongly agree, with 13 statements (items). The reliability coefficient of Cronbach's alpha was α=0.878.

In the second part, several interesting findings and important points for this research were noted. The statement "I like geometry in math class" was agreed by 66%, while only about 30% said they were familiar with dynamic geometry software such as GeoGebra, Sketchpad, etc. The statement "Learning geometry on the computer motivates me to explore similar content in other courses" was agreed by 67%, and 78% of students confirmed that learning geometry on the computer allows them to understand different concepts and connect math to other courses. In addition, a large majority of students, over 90%, responded that they can visualize concepts and theories when the professor teaches subject courses and that they can relate problem-solving tasks to their subject area. They also indicated that they can visually visualize formulas and equations when studying for individual courses.

A discouraging finding was that only 3 students had checked out at least 3 or more books from the library, 7 had checked out less than 3 books, and 34 students had not checked out any books or additional literature from the library to pass some courses. When given the opportunity to select multiple responses, 9 students responded that they used textbooks to study and pass courses, while the vast majority of 44 students used lecture presentations, 39 used lecture notes, and 29 used lecture notes from other students.

There are several possible measures and strategies that can be taken to improve students' interest and ability to learn mathematics, and these are just some of them: connecting with real-life examples where they can see the practical application of mathematical concepts, using different teaching methods, a personalized approach, a positive learning environment through conversation and collaboration, using modern technology, and continuous support, praise and encouragement, as well as regular feedback on students' personal progress.

5 CONCLUSIONS

This paper explains the importance of vector visualization, computational operations with vectors and the application of vectors in mechanical engineering. In the theoretical part, the basic terms related to vectors are defined and their basic properties and operations where we often encounter multidimensional problems are explained. Van Hiele's theory is also described, which is a very important basis in the geometric consideration of mathematical concepts. Through several examples related to the courses that students study at professional studies, it was shown that in mechanics vectors are used to describe forces, velocities and accelerations of bodies, in statics it is shown how vectors are used to analyze the balance of a system, in the science of solidity they are also used to stress and deformation analysis. On the other hand, a short survey was conducted with first-year students who study vectors in several courses and connect them with learned mathematical concepts. They also expressed their views that it is important for them to have a picture, an idea, a graph, etc. when learning, which will make it easier for them to understand and follow the theory. However, most students very little or almost never borrow additional books and literature for studying and passing professional courses, where they might see additional geometric and visual concepts that they could later use in practice. For further research, it would be possible to compare the attitudes of these same students and their success on their written exams, where the visualization of certain concepts and connections with the theoretical part would be particularly emphasized. The aim of teaching mathematics is to create mathematically literate people who can apply the knowledge they have acquired in certain life situations, which certainly always have a problematic outcome. The ability to develop logical thinking and reasoning, as well as the connection of mathematics with other disciplines and the use of visualizations contribute significantly to changing the concept of mathematics from a traditional approach to a modern way of learning and teaching.

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CONTACTS:

Maja Culetic Condric, professor of mathematics and physics, senior lecturer,
University of Slavonski Brod, Technical department,
Ulica 108. brigade ZNG, 35 000 Slavonski Brod, Croatia, mccondric@unisb.hr