

# OPTIMIZATION OF ENERGY BY ROBOT MOVEMENT

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This paper is describing the influences of movement parameters on energy consumption during robot operation. The experimental verification of mathematical models was performed in the laboratory of the Institute of Production Machines, Systems and Robotics at Brno University of Technology. The measure results are arranged in tables and diagrams from which final evaluation for praxis follow. One of them is the outline of movement parameters setting from the side of robot users before the program is put in manufacture. Another one is recommendation for robot producers to append new items to driving system menu referring to energy consumption calculator for off-hand programming as well as energy consumption meter.

## Keywords

Robot, Motion, Energy Consumption, Measuring, Velocity, Acceleration

## 1. Introduction

One of the aspects by industrial robots and manipulators development is their motion improvement which can include many parameters as maximal average speed, optimal path shape, minimal time needed for target point achievement, elimination of swift or even shock speed and acceleration changes by drive initializing, optimal changes of acceleration, economical consumption of energy as well as combination of parameters mentioned.

The subject of this paper is analysis of energy consumption in dependence on movement parameters with specialization in optimal speed, acceleration and jerk.

## 2. Optimal time, speed and acceleration

Force effects can be evaluated either according to the path or to the time during which the force affects. In this case we look for optimal speed of steady linear movement and optimal acceleration of steadily accelerated movement. Therefore we must use following formula (1) for energy calculation:

$$W = \int_{s_1}^{s_2} F(s)ds \quad (J) \quad (1)$$

By robot movement loss of energy can appear because of impacts and sudden changes of velocity and acceleration that cause framework vibration or deformation, because of friction, thermic losses etc. The following calculation assigns energy needed for an object manipulation done by the robot over the base.

It is necessary to assign some simplifications.

- object is moved along given linear path with steady linear or steadily accelerated / decelerated movement
- object is carried over the base through the air
- friction forces are not evaluated
- we evaluate the friction force of ambience

$$F_o = \frac{1}{2} CS \rho v^2 = Kv^2 \quad (2)$$

- control unit input is constant

$$P = UI \quad (3)$$

the other influences are disregarded including energy consumed for robot components movement – there is a large amount of robot configurations and measurement can be arranged so that we are able to evaluate exactly the energy consumed for these conditions given.

### 2.1 Steady linear movement

The total energy is sum of energy needed for moving the object, energy for supply of driving unit and energy for the friction force of ambience (4).

$$dW = Fvdt + UIdt + F_o vdt \quad (4)$$

The friction force of ambience is deduced from the Bernoulli equation (2) where for given object shape and size and for given ambience the constant K can be simplification (5).

$$F_o = \frac{1}{2} CS \rho v^2 = Kv^2 \quad (5)$$

For steady linear movement acceleration is zero, velocity is constant and the movement starts with zero velocity and zero initial consumed energy.

$$\begin{aligned} W &= \int_{t_0}^t Fvdt + \int_{t_0}^t UIdt + \int_{t_0}^t Kv^3 dt \\ t_0 &= 0, \quad v_{t_0} = 0, \quad W_{t_0} = 0 \Rightarrow C = 0 \\ W &= Fvt + Ult + Kv^3 t \end{aligned} \quad (6)$$

$$\begin{aligned} W &= Fvt + Ult + Kv^3 t \\ v &= \frac{s}{t}, \quad UI = P \quad F = ma, \quad a = 0 \Rightarrow F = 0 \\ W &= Fvt + Pt + K \frac{s^3}{t^3} t \end{aligned} \quad (7)$$

After substitutions and modifications we can get from the equation (6) the equation (7).

For this type of movement there are only present the driving system supply and the force necessary for overcoming of ambience friction (8):

$$W = Pt + K \frac{s^3}{t^2} \quad (8)$$

The optimal time for movement can be found in the point where the first derivation equals zero because there we can find the local extreme and after that we put its value (9) into the second derivation. The positive or negative value of its result shows to local minimum or maximum.

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial t} (Pt + Ks^3 t^{-2}) = 0$$

The local extreme of this function can be found:

$$\begin{aligned} P + Ks^3(-2t^{-3}) &= 0 \\ P - 2Ks^3 t^{-3} &= 0 \\ t^{-3} &= \frac{P}{2Ks^3} \\ t &= s \sqrt[3]{\frac{2K}{P}} \end{aligned} \quad (9)$$

By calculating the second derivation according time and making substitution of variable  $t$  we can find out that there is local minimum (10):

$$\begin{aligned} \frac{\partial^2 W}{\partial t^2} &= \frac{\partial}{\partial t^2} (Pt + K \frac{s^3}{t^2}) = \frac{\partial}{\partial t} (P - 2Ks^3 t^{-3}) \\ \frac{\partial^2 W}{\partial t^2} &= -2Ks^3(-3t^{-4}) = 6Ks^3 t^{-4} > 0 \\ t &= s \sqrt[3]{\frac{2K}{P}} > 0 \end{aligned} \quad (10)$$

In this way we can say that for the equation of energy consumption in the form (11)

$$dW = Fvdt + Uldt + F_0 vdt$$

$$W = Pt + K \frac{s^3}{t^2}$$
(11)

the local minimum of energy consumed for object movement in dependence on the time of movement can be found. That also means that there exists minimum velocity that causes minimum energy used by robot movement.

## 2.2 Steadily accelerated movement

Similar calculation can be performed for steadily accelerated movement (12). Simplifications are the same as for steady linear movement.

$$dW = Fvdt + Uldt + F_0 vdt$$

$$F_0 = \frac{1}{2} CS \rho v^2 = Kv^2$$

$$W = \int_{t_0}^t Fvdt + \int_{t_0}^t Uldt + \int_{t_0}^t Kv^3 dt$$

$$v = f(t) \quad v = at, \quad a = \text{konst}$$
(12)

For steadily accelerated movement acceleration is constant and the same is valid for the force F (14).

$$W = \int_{t_0}^t Fatdt + \int_{t_0}^t Uldt + \int_{t_0}^t Ka^3 t^3 dt$$

$$W = Fa \frac{t^2}{2} + Ult + Ka^3 \frac{t^4}{4} + C$$

$$t_0 = 0, \quad v_{t_0} = 0, \quad W_{t_0} = 0 \rightarrow C = 0$$

$$W = ma^2 \frac{t^2}{2} + Pt + Ka^3 \frac{t^4}{4}$$
(13)
(14)

The equation (14) was calculated for energy consumption by manipulation with an object for steadily accelerated movement.

We are looking for optimal time of movement from the point of view of energy consumption. Therefore we must find where its first derivation equals zero (15) which is the local extreme of this function and after that we put its value into the second derivation. The positive or negative value of its result shows to local minimum or maximum similar as it was shown above.

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial t} (ma^2 \frac{t^2}{2} + Pt + Ka^3 \frac{t^4}{4}) = 0$$
(15)

$$\frac{\partial W}{\partial t} = \frac{ma^2}{2} 2t + P + Ka^3 4t^3 = 0$$
(16)

$$Ka^3 t^3 + ma^2 t + P = 0$$
(17)

This function (17) has two imaginary roots that are not reflected and one real root – solved with help of program Mathematica5 (18).

$$t \rightarrow - \frac{\left(\frac{2}{3}\right)^{1/3} ma^2}{\left(-9 Ka^6 P + \sqrt{3} \sqrt{4 Ka^9 ma^6 + 27 Ka^{12} P^2}\right)^{1/3}}$$

$$\left. \frac{\left(-9 Ka^6 P + \sqrt{3} \sqrt{4 Ka^9 ma^6 + 27 Ka^{12} P^2}\right)^{1/3}}{2^{1/3} 3^{2/3} Ka^3} \right\}$$
(18)

The equation of energy consumption by robot movement and manipulation with an object for steadily accelerated movement in the form of 16):

$$W = ma^2 \frac{t^2}{2} + Pt + Ka^3 \frac{t^4}{4}$$

This equation has local extreme (19) from which we can find out if it is local minimum or local maximum which help of substitution of  $t$  value (18) into the second derivation (19) of the function:

$$\frac{\partial^2 W}{\partial t^2} = \frac{\partial^2}{\partial t^2} (ma^2 \frac{t^2}{2} + Pt + Ka^3 \frac{t^4}{4}) = \frac{\partial}{\partial t} (ma^2 t + P + Ka^3 t^3)$$

$$\frac{\partial^2 W}{\partial t^2} = ma^2 + Ka^3 t^2 > 0$$
(19)

Because both parts of the inequality are bigger than zero:

$$ma^2 > 0 \quad Ka^3 t^2 > 0 \quad K = \frac{1}{2} CS \rho > 0$$

$a^3 > 0$  for accelerated movement  $t^2 > 0$

It is not necessary to substitute the value counted with help of the program Mathematica5 (18) into the equation (19) because for all P, K, a and M the root of this equation takes positive value.

### Result:

There exists optimal time of movement for given path by steady linear and also steadily accelerated movement where minimum energy is needed for operation. Therefore there also exists optimal speed respective optimal acceleration for the same types of movement from the point of view of energy consumption.

## 3. Experiment implementation

The mathematical model mentioned above was verified with help of the robot type IRB 4400/60 which is placed in the laboratory of the Institute of Production Machines, Systems and Robotics at Brno University of Technology. Both the orientation and the positioning systems consist of three rotary kinematic pairs, so there are six controlled axes at disposal. Anthropomorphic working space is enlarged by use of track motion.

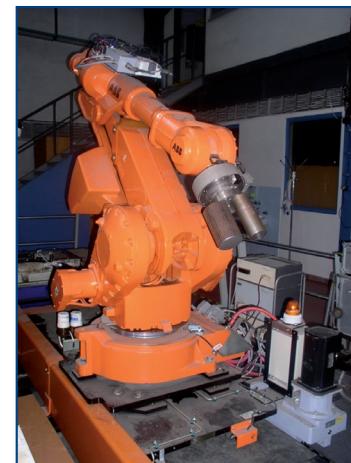


Figure 1. Robot IRB 4400/60

Some of robot IBR 4400/60 basic parameters	
Dimensions of base/mass	920x640 mm/980 kg
Maximal load/range of the 5 <sup>th</sup> axis vertical	60 kg / 2140 mm
Max.TCP (Tool Centre Point) speed/acceleration	2.2 ms <sup>-1</sup> / 12 – 14 ms <sup>-2</sup>
Repeated positioning accuracy	0.07 – 0.1 mm

For programming the technological language RAPID was used. It is designed for programming on high level and is oriented to the user. It includes instructions for program running control, setting of movement, operating the input and output signals, arithmetic and logical expressions etc.

A holder for quick exchange of weight was fastened to the wrist of the robot. The measuring was always realized with 0, 5, 10, 15 and 20 kg weight. Higher loading would shorten the length of TCP path during measuring which results from loading diagram valid for this robot.

Electric energy consumption was measured with digital wattmeter Yokogawa WT 1600 S. It is a four-canal wattmeter that makes possible measuring of voltage, current, output and electrical energy consump-

tion on base of instant values integration measured with high frequency. The measure results were gained with the period of 200 ms. The first three canals were used for measuring of input and energy consumption and the fourth canal was used as the voltmeter for the output signal connected over connection plate for I/O signals of the control unit. The output signal was set to logical 1 or logical 0 with help of programming instruction with the name Mereni\_On. After its adjustment to logical 1 the voltage value was increased to 24 V as the signal for start-up the energy consumption measuring.

During measuring the temperature of motors was found out with infrared thermometer IR60 8A which works in the range of 0 – 400°C with accuracy 2%. The measuring of motor temperature was only performed on the cover of the motors because of technical reasons. The aim of this measuring was to assure the fixed temperature of motors so that we have comparable measure results. The motors had to have stabilized their working temperature.



**Figure 2.** Digital wattmeter Yokogawa WT 1600 S

The temperature of surroundings was controlled with digital thermometer LM 91 HT but it was only controlled as informative value. All measuring was performed during summer months when temperatures in the workroom were around 25 – 27°C.

#### 4. Measure results

There was single measuring performed with the speed of  $v = 100 - 500 - 700 - 1000 - 1500$  mm/s without statistical evaluation, each measuring only three times. In this way the zone between 700 – 1000 mm/s with minimum consumption was found out. After that energy consumption was measured for each point of the graph 10 times with the weight 0 to 20 kg with statistical evaluation. The evaluation seems to be comparatively exact compared with the values of deflections. The results were interpreted with help of the program Excel.

##### 4. 1 Speed influence

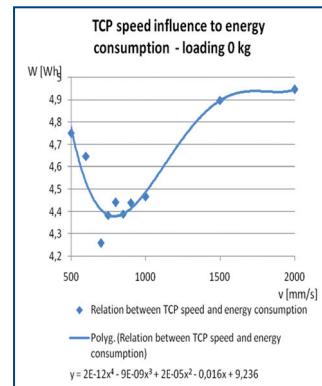
The path selected for this measure was 500 mm in y axis in the framework of the base. The programs used for the purposes of measuring had common parts concerning:

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PROC main(); MoveJ a0, v800, fine, tool 0

Reset Mereni_ON	setting up of output signal
WaitTime 2	waiting time
Set Mereni_ON	start-up of measuring
FOR i FROM 1 TO 10 DO the measured part of program; ENDFOR	
Reset Mereni_ON	end of measuring
WaitTime 2	waiting time
Stop; ENDPROC	[ABB Robotics 1999]

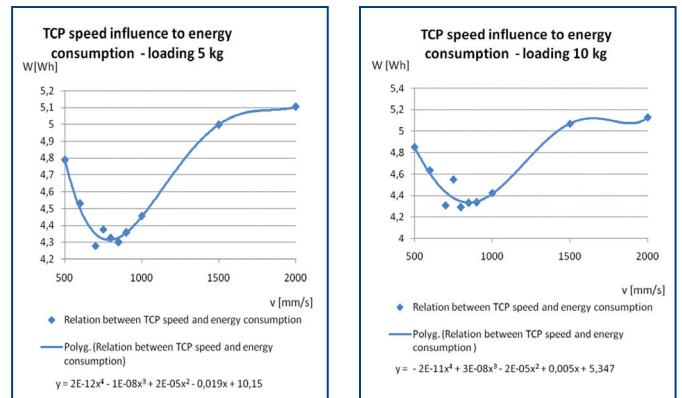
The following five graphs show the results of measuring in axis y with the load of 0 to 20 kg. The points are chained with help of regression curve in form of polygon with the fourth degree. This degree was the lowest one which could be used. The equations of these polygons were computed by program Excel.



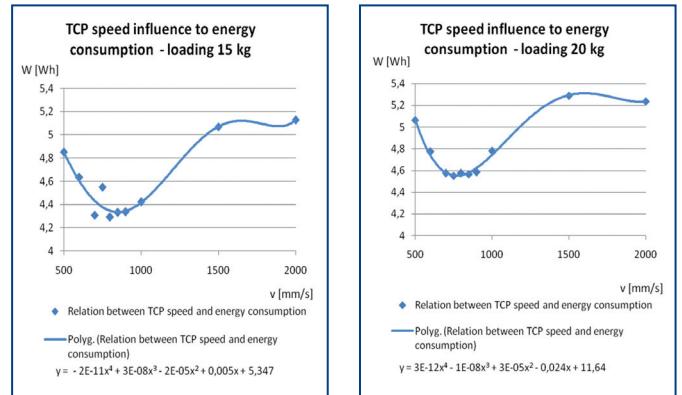
v [mm/s]	W [Wh]	$\sigma^2$	$\sigma$ [Wh]
500	4,571	7,01269E-05	0,0084
600	4,648	0,001815715	0,0426
700	4,259	0,000370685	0,0193
750	4,384	0,000552838	0,0235
800	4,442	0,000495916	0,0223
850	4,390	0,000479619	0,0219
900	4,441	0,000252506	0,0159
1000	4,467	0,000491718	0,0222
1500	4,898	0,003568125	0,0597
2000	4,947	0,002948713	0,0543

**Figure 3.** Graph and table TCP speed influence to energy consumption with loading 0 kg

The other graphs are included without their tables because of lack of space. In the graphs the existence of optimum speed in relation with the consumption of energy by robot operation can be seen.



**Figure 4.** TCP speed influence to energy consumption with loading 5 and 10 kg



**Figure 5.** TCP speed influence to energy consumption with loading 15 and 20 kg

## 4.2 Acceleration influence

The drives run during starting with maximal acceleration that comes out from given movement, loading and maximal output of drives. The acceleration can be reduced with function AccSet which has two parameters setting up the reduction of maximal acceleration and the value jerk.

AccSet 10,100 keeps 100% jerk and reduces acceleration to 10% of maximal value while AccSet 100, 10 sets up the acceleration to maximal value and leaves jerk on 10% of available maximum. By higher speed it is often necessary to reduce acceleration because of manipulation with fragile objects.

For this measure steady linear movement in axis x in the framework of the base with length 1 000 mm was chosen, TCP speed was  $v = 2000$  mm/s. With help of the function AccSet the maximum acceleration was reduced to 10–20–30–40–50–60–70–80–90% of maximal available one. The following five graphs show the results of measuring with the load of 0 to 20 kg. The points are also chained with help of regression curve in form of polygon with the degree number four which were computed by program Excel.

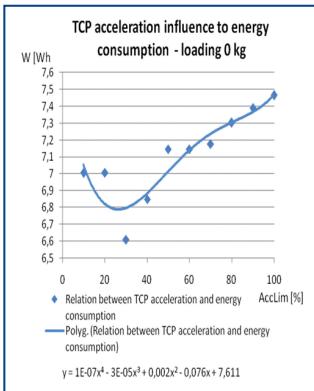


Figure 6. Graph and table acceleration reduction influence to energy consumption with loading 0 kg

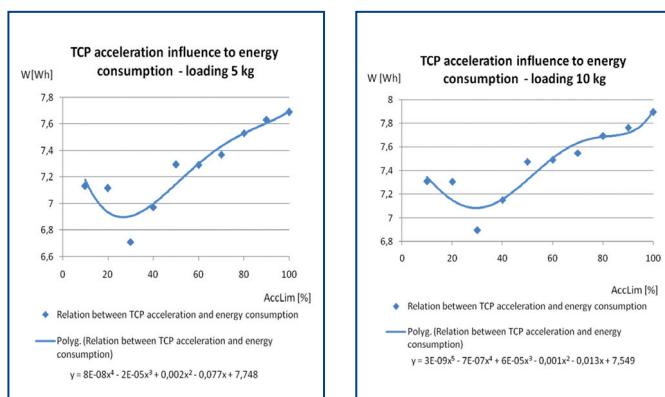


Figure 7. Acceleration reduction influence to energy consumption with loading 5 and 10 kg

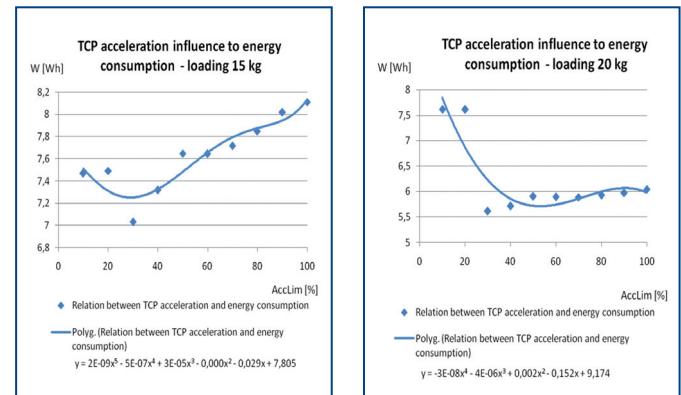
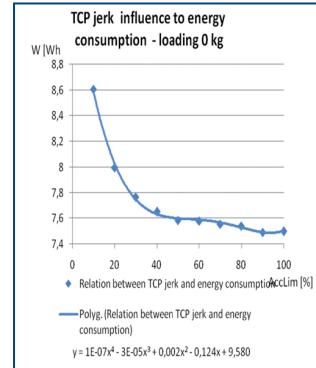


Figure 8. Acceleration reduction influence to energy consumption with loading 15 and 20 kg

The minimum energy consumption was in all cases by a = 30% of maximal available value. The other graphs are included without their tables because of lack of space. The conclusion of all these results is that there exists optimum reduction of acceleration and its use reduces the consumption of energy by robot operation.



Jerk [%]	W [Wh]	$\sigma^2$	$\sigma$ [Wh]
10	8,607	1,34596E-05	0,0037
20	7,997	0,000119215	0,0109
30	7,766	8,05051E-05	0,0090
40	7,652	0,000182585	0,0135
50	7,586	0,000182427	0,0135
60	7,581	0,000197693	0,0141
70	7,556	0,000212808	0,0146
80	7,541	6,045E-05	0,0078
90	7,492	0,000215105	0,0147
100	7,500	0,016029311	0,0160

Figure 9. Jerk reduction influence to energy consumption with loading 0 kg

## 4.3 Jerk influence

Jerk works similar as acceleration in the stage of starting movement which means that the robot operates with maximal possible change of acceleration. During verifying the jerk influence to energy consumption the movement in axis x in the framework of the base with length 1 000 mm was used, TCP speed  $v = 2 000$  mm/s. The setting was done with help of the function AddSet similar as by acceleration influence measuring. The jerk value was reduced to 10–20–30–40–50–60–70–80–90% of maximal available one.

The measure results were similar when the loadings from 0 to 20 kg were used. Any limitation of jerk does not lead to decrease of energy consumption. This value can be limited to prevent impacts till about 50 % maximal available value. Higher limitation causes distinctive increase of energy consumption.

## **5 Proposal of method for adjustment the movement parameters**

The aim of this paper was among others the proposal of a simple and quick method of adjustment the movement parameters for robot operation to optimize energy consumption. The method comes out from the possibilities of the control unit menu and the programming possibilities of the programming language.

During programming the speed of individual movements can be set up parametrically which means their definition in the head of the program. In this way the values of speed can be easily changed in one place. After that energy can be orientationally measured for a limited number of TCP speed values to find the zone that is interesting from the point of energy consumption. Then more detailed measuring can be performed.

The proceeding can be recapitulated as follows:

- Compilation of a technological program
- Declaration of speed parametrically in the head of the program
- Disposable tentative measuring of a limited number of TCP speed
- Choice of the zone with lower energy consumption
- Detailed measuring in this zone and evaluation with help of accessible software
- Adjustment the optimum acceleration for this optimum speed – in similar way

The measure results can be evaluated including the regression curve and computing its minimum with help of one of the mathematical programs – Excel, MathCad, Matlab etc.

## **6 Conclusions**

In this paper the existence of optimal working speed and optimal acceleration by robot operation was shown. The monitoring of energy consumption can be important in the cases of limited energy sources for example by research of distant areas. In such conditions functionality can be distinctly extended by adjusting the correct movement parameters. However these results cannot be used if the pri-

or target is minimal production time without consideration of production costs.

The robot producers can help the users by implementation new items to driving system menu with energy consumption calculator for off-hand programming as well as energy consumption meter for testing programs from the point of view of energy consumption. Similar calculator already exists for time calculation of programmed movements in the programming language Rapid [ABB Robotics 1999].

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