OPTIMIZATION OF WORKERS QUANTITY USING MATHEMATICAL MODEL

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Production and maintenance processes are inherent in the life cycle of every product. Despite great efforts to automate these processes, a great deal of human resources are still required, which represent a significant part of the financial costs. Each process is composed of sub-tasks that require certain specifics in terms of the number of staff, their expertise, qualifications and experience. It is assumed that the staff are divided according to specifics into different groups with differing wages. Workers' wages are reflected in the final financial cost of the product, its life cycle and its return. Reducing labour costs in a production or maintenance process can be achieved by reducing the total number of staff deployed in the process and by appropriately composing groups of workers. Reducing labour costs leads to increased competitiveness in the market. The main tools of competitiveness are price, speed and range of services offered. This paper examines a strategy that uses price as the main tool for competitiveness in the market. One way to reduce the final price of the product for the customer is to optimise the costs of human resources. This can be achieved through appropriate planning of staff shifts. The specifics of the deployment of staff in a production or maintenance process depend on the requirements of the process sub-tasks. This means that each group of workers can only handle a certain group of tasks according to their qualifications. A Binary Programming Problem with Linear Bonds will be used to plan the deployment of staff, aiming to minimize the number of workers needed in a production or maintenance process within a predefined timeframe.

KEYWORDS

Maintenance, Mathematical model, Binary Programming Problem, Scheduling, Crew members

1 INTRODUCTION

The world of air transport is a very specific industry, where great emphasis is placed on safety, speed, precision and comfort. Due to these characteristics, air transport is considered the top commercial mode of transporting both passengers and goods. However, these advantages are offset by the high cost of acquiring and operating the resources and equipment. One of the reasons for the high costs are the high safety standards. Ensuring safety depends not only on the design of the system but also on regular and consistent maintenance, which is linked to human resources.

The current development of technology allows a large part of the aircraft production process to be automated, but unfortunately the maintenance process is still tied to human resources. The expected service life of the aircraft and the cost of maintaining the aircraft in an airworthy condition are already taken into account during its design. Aircraft manufacturers try to minimize the life cycle costs of the aircraft by choosing the appropriate materials, design, aircraft layout and minimizing the training of staff. Other parameters that enter into the design of the aircraft also include the maintenance demands, which focuses on the amount of consumables, tools, spare parts used, the complexity of technological procedures and the number of maintenance staff. Each manufacturer has its own standards for maintenance requirements, which form its corporate strategy. The main standards are the use of uniform consumables, tools, similarity of technological procedures and the effort to comply with the professional requirements for maintenance staff and their number. Throughout the history of aviation, various aircraft designs have been tried that were advantageous in terms of operational parameters but challenging to maintain both technically and in terms of staff. One example is the location of the power units high above the work area, where it was both challenging and risky for maintenance workers to carry out maintenance and other tasks at such heights. For this reason, maintenance requirements have also been introduced into the development of aircraft design and technology, particularly to increase the safety-related maintenance intervals and to reduce the requirements for the expertise and number of maintenance staff. The overall requirements for the expertise and number of maintenance staff to perform aircraft maintenance is determined by the type of aircraft, its configuration, the level of maintenance, the technical condition, the environment in which the maintenance is performed, along with the maintenance strategy of the aircraft operator and the staffing structure of the aircraft maintenance organisation.

In accordance with the safety requirements and the manufacturer's decision, a aircraft maintenance programme of the aircraft is established by the manufacturer and approved by the competent authority to prevent malfunctions from occurring during operation and endangering the safety of passengers or bystanders. The aircraft maintenance programme defines the activities and their intervals. These intervals are set according to the safe limits of the service life of components. The regularity of maintenance is related to the life cycle of the aircraft and its specific components, flight hours, number of take-offs, seasonality and other manufacturer-defined constraints, if any. The maintenance procedure is prescribed in task cards that are linked to the type, version and configuration of the aircraft.

The task cards contain the prescribed aircraft maintenance technician competency category that the maintenance technician must meet to verify the performance of the work procedure. According to the work procedure, it is possible to access the expected staffing requirements for the performance of the task card. Staff shall obtain the necessary category of competence upon completion of the aircraft mechanic training regulated by the competent administrative authority. The training of aircraft mechanics takes place in specialised training centres.

Two types of maintenance are combined in the aircraft's scheduled maintenance. The first type of maintenance is line maintenance, and the second type is so-called heavy maintenance. Line maintenance consists of performing simple inspection activities between scheduled aircraft flights in designated areas of the airport. The fact that line maintenance is carried out in between flights puts pressure on the completion of such maintenance on time. Line maintenance deadlines are very strict, and its precision requires maintenance to be planned literally to the minute as it is subject to the flight schedule. For the above reasons, it is essential to have a sufficient number of line maintenance staff to avoid delays to flights. At the same time, the surplus of maintenance staff on the apron leads to an increase in staff wage costs, the difficulty of coordinating the maintenance process, and the risk of incidents occurring. Therefore, the number of line maintenance staff for planned line maintenance must be properly arranged in advance according to the scope of the planned maintenance. Line maintenance staff shall hold a Category A Aircraft Maintenance Technician Certificate of Competency to enable the issue of a valid certificate of release to service after performance of line maintenance. In the event of finding an unsatisfactory condition during line maintenance, minor damage is repaired or the recommended interval when the aircraft must be subjected to heavy maintenance by an aircraft maintenance, repair and overhaul organisation may be reduced. The decision to perform heavy maintenance depends primarily on the technical condition of the aircraft, the severity of the finding, safety standards and the decision of the operator. During line maintenance, some task cards that do not affect airworthiness may be deferred to heavy maintenance. However, this situation is not desirable, so it is the intention of most aircraft operators to carry out line maintenance activities in a timely manner.

The current trend amongst aircraft operators is to perform the less demanding task cards in both types of maintenance so that most maintenance is performed during the downtime between flights. Aircraft operators include some of the less demanding task cards of the aircraft maintenance programme in the line maintenance schedule, thereby reducing the time an aircraft is in heavy maintenance and reducing the staffing requirements to perform heavy maintenance. In the course of line maintenance, on the other hand, a part of the activities that do not limit the airworthiness of the aircraft is deferred to heavy maintenance. With these capabilities, aircraft operators can appropriately schedule aircraft maintenance based on schedule deployments or wear and tear, thereby increasing the efficiency of the aircraft's life cycle. This option allows aircraft operators to select a maintenance strategy within the capabilities of the aircraft maintenance programme. Maintenance strategy planning enables the management of overall maintenance costs and planning of the aircraft's periods of operation. One of the main parameters here are the staffing requirements to ensure regular maintenance and the time the aircraft spends in maintenance.

Heavy maintenance is carried out by aircraft maintenance, repair and overhaul organisation at regular intervals or according to the technical condition of the aircraft. During heavy maintenance, an aircraft is subjected to more demanding activities by an aircraft maintenance, repair and overhaul organisation, the purpose of which is to subject the aircraft to a more detailed examination of its technical condition and to perform technologically demanding tasks or modifications. The most frequent requirements from aircraft operators are the performance of routine maintenance, exit checks of the technical condition of the aircraft after the lease contract is terminated and aircraft modifications. The working area in an aircraft maintenance, repair and overhaul organisation is usually divided into production lines and workshops with different focuses. Depending on the requirements of the aircraft operator, the aircraft is assigned to the appropriate production line where maintenance and workshop staff perform maintenance.

As mentioned above, the difficulty of heavy maintenance varies according to the maintenance strategy chosen by the aircraft operator, the extent of maintenance, the type of aircraft and its technical condition. For these reasons, the requirements for maintenance staff are not always the same. In general, heavy aircraft maintenance is more personnel-intensive, and there are also more groups of specialised maintenance staff. Heavy maintenance staff must hold a category B1, B2 or C aircraft maintenance technician certificate. Category B1 qualification allows the holder to issue a certificate of release to service after maintenance has been carried out on the airframe, power unit, mechanical and electrical systems. At the same time, the category B1 rating includes category A. The category B2 rating permits the holder to issue a certificate of release to service after maintenance of avionics and electrical systems but does not include any other category authorisation. The category C rating allows the holder to issue a certificate of release to service after completion of maintenance at an aircraft maintenance, repair and overhaul organisation. Outside of these categories, each group of personnel in an aircraft maintenance, repair and overhaul organisation has a designated priority focus on a specific area of the aircraft or its system. Production line and workshop staff tend to include test technicians, plumbers, painters and other specific groups of workers.

The number of staff deployed to carry out the maintenance of the aircraft is critical in determining the duration maintenance and the final cost incurred for the maintenance of the aircraft. These costs are crucial for the aircraft maintenance, repair and overhaul organisation that employs the workers, as based on this their services are offered on the market. The aim of an aircraft maintenance, repair and overhaul organisation is to offer its services at the lowest possible price in order to be competitive on the market. Solving the issue of manpower brings the possibility to minimize the staffing requirements for aircraft maintenance. The number of heavy maintenance personnel is usually determined based on the scope of the planned work, the complexity of the maintenance in relation to the number of estimated standard hours, legislative requirements, the space on the work area and the required completion date. Depending on these parameters, the recommended number of personnel in each group for the staffing of aircraft maintenance is determined in the aircraft maintenance, repair and overhaul organisation.

During their life cycle, aircraft may be in service, decommissioned or undergoing maintenance. In the course of operation, aircraft are subject to line maintenance, which requires the activity of maintenance personnel on the airport apron.

When the aircraft is parked, it is parked in the parking areas of the airport where the aircraft can be kept on standby or long-term standing condition. The difference between these conditions is the level of preservation of the whole aircraft and the readiness of its systems for flying. Also, when the aircraft is shut down, it is necessary to subject it to routine maintenance activities for monitoring of its technical condition. The condition of the parked aircraft indicates the intensity of inspections and the maintenance costs incurred. Therefore, the ongoing trend is to move aircraft to a dry environment at a constant temperature outside the peak shutdown season to reduce the cost of recommissioning and the risk of corrosion arising. The operator should only withdraw the aircraft as a last resort if it does not find a use for the aircraft in the following period.

Aircraft are sent to aircraft maintenance, repair and overhaul organisation for the purpose of heavy maintenance. Heavy maintenance can take from several days to weeks. During heavy maintenance it is not possible to operate the aircraft, so the operator does not profit from it. It is in the general interest of all aircraft operators to keep the aircraft's stay with the aircraft maintenance, repair and overhaul organisation as short as possible, as this stay reduces the return on the costs incurred in acquiring and operating the aircraft. The length of time an aircraft remains in heavy maintenance depends on the extent and complexity of the contents of the task cards, the technical condition of the aircraft and the number of maintenance staff. The set of task cards to be performed in heavy maintenance is created by the Continuing Airworthiness Management Organisation (CAMO). The CAMO department monitors the current condition of the aircraft and based on the aircraft maintenance programme, proposes task cards to be carried out in due time so that the aircraft's airworthiness is not revoked.

The work package consists of routine, non-routine, technological and administrative task cards. Routine task cards are set by the aircraft manufacturer according to the aircraft maintenance programme. Non-routine task cards are created by administrative order, modification requests or fault corrections. The technology task cards represent activities associated with the preparation of the aircraft for maintenance and the administrative task cards represent administrative tasks and records of the internal processes of the aircraft maintenance, repair and overhaul organisation. A work package is created before maintenance begins and is modified as needed during maintenance. This makes maintenance planning more complex and challenging to coordinate throughout the maintenance process. To this end, a production planning department has been set up to react to changes in the task card set and to take appropriate action to meet the scheduled release date. Meeting the maintenance completion date depends primarily on maintenance staffing, availability of consumables and spare parts, appropriate timing during maintenance and coordination of the entire maintenance process.

The work package for the entire project can be divided into phases, with each phase of the project having its own specific characteristics. First, the technological task cards are opened in preparation of the aircraft for maintenance and then tests and checks of the technical condition are carried out. This is the moment when the decision is made to set a date for the completion of the maintenance. According to this term, staff are assigned to repair technical defects and make modifications. Just before the maintenance is completed, final tests and checks of the work carried out are performed. At the end of the maintenance, a certificate of commissioning after maintenance has been carried out is issued and, if necessary, a test flight of the aircraft is implemented. After this the aircraft is handed back to the aircraft operator.

The set of routine task cards can be categorised according to the different levels of maintenance, which are divided between line maintenance and heavy maintenance.

For line maintenance, there are transit, daily and A checks as the designated maintenance levels. These maintenance levels can be carried out within a few hours, usually on the apron during the period between scheduled flights. Heavy maintenance is the performance of B, C, and D checks, where the aircraft is subjected to a more thorough examination of its technical condition in the covered hangars of an aircraft maintenance, repair and overhaul organisation.

In heavy maintenance, B check is the least demanding. This check includes routine task cards performed at 8 month intervals or on a 500 flight hour cycle. B check can be completed in one to five days and includes routine task cards to check the function of systems and components without disassembling parts of the aircraft.

C Check is more demanding. This revision includes routine task cards performed at intervals of 15 to 24 months or on a cycle of 3000 flight hours. The C check can be performed within two weeks during which most parts of the aircraft are inspected. This check includes a general overhaul of the aircraft's engines and systems.

The most thorough is D check. D check is performed every 6 years or 20,000 flight hours. This check may take longer than 2 months depending on the condition of the aircraft. Routine task cards in D check include complete structural inspections, component overhauls and engine replacements.

At present, A, B, C and D checks are not further defined by the aircraft manufacturers but are indirectly defined by the performance interval of each task card. Maintenance levels are still used by aircraft operators to categorise aircraft maintenance orders in this way, as similarities can be found between them, and it will make it easier to describe the nature of the maintenance. The reasons for limiting the use of maintenance levels by manufacturers are the variety of task card requirements, the heterogeneity of the condition of individual components and the possibility of continuous maintenance between flights.

The restriction on the use of maintenance levels by the manufacturer is due to situations where a planned B check inspection on a particular aircraft should take 9 days, but due to the longer time and content, will be designated by the operator as a lighter C check. Additionally, in this situation there will also be a different list of basic task cards that does not match either of the B or C checks.

The aforementioned maintenance levels require adequate maintenance staffing. Depending on the level of aircraft maintenance, the number of deployed personnel increases proportionally, and the range of staff specializations likewise expands.

The strategy of an aircraft maintenance, repair and overhaul organisation may focus only on selected maintenance levels or only on a narrow range of activities, which may be, for example, modification of passenger cabins. By regulating the number of personnel deployed, it is possible to control the total time of the aircraft in maintenance, but only up to the permissible limit, which is determined by the execution of time-consuming technological procedures. In determining the recommended number of maintenance staff per work area, it is necessary to take into account the change in the cost of employee salaries, the current situation in aviation, the current situation in the labour market, the necessary implementation of changes in the structure of the aircraft maintenance, repair and overhaul organisation and the workload of managers. By managing staffing requirements during ongoing maintenance, the workload of the staff can be balanced while stabilizing daily results. Stabilizing daily results provides an additional opportunity to more easily identify bottlenecks in processes, enable more accurate planning, increase efficiency of work through all departments of the aircraft maintenance, repair and overhaul organisation and increase the company's market potential.

2 CURRENT STATE OF KNOWLEDGE

As it is generally known, in most companies human resources is the expense with the highest cost. That is why it is also an effort to optimize the use of employees so that their involvement in the work process is as efficient as possible. The issue of planning and optimization of human resources in enterprises is dealt with in a number of scientific studies. A basic overview of mathematical models and management studies dealing with human resource planning is presented in a review article [de Bruecker 2015], in which the authors focus mainly on publications written after 2000. The authors also compare the mathematical models with management studies and try to make recommendations to support the development of new and more realistic planning techniques. The summary of methods used for planning and assigning workers on production lines is also the subject of a review article [Dolgui 2019], which focuses on such methods that allow flexible reconfiguration of a production line, for example, to a different product or a different production component, based on a suitably selected combination and quantity of workers. This overview includes publications from 1998 to 2019.

In the article [Battaïa 2015], the authors focus on determining the optimal number of employees for the introduction of a new assembly line for the production of different types of products. To solve this problem, the authors present an integer linear mathematical model. Since this is an NP-hard problem, the mathematical model itself is part of the heuristic, which allows a faster solution to be found. A similar problem, i.e., determining the optimal number of workers on a production line using an integer linear programming model, is solved by the authors of the paper [Delorme 2019]. For the solution, the authors use a heuristic method consisting of designing their own algorithm for the solution procedure using dynamic programming. The authors of the paper [Liu 2019] also present their approach to determining the optimal number of workers on an assembly line. The authors of the paper [First 2016] build on the work published in the paper [Feillet 2010], which deals with the assignment of employees with multiple skills to a work schedule. The authors present a modified integer mathematical model, for the solution of which they propose to use the Branch and Price method in contrast to the original solution, where the simplex method was used. Thanks to the modifications, the authors achieved more than twice the computational capacity in a much smaller time interval. The scheduling of multi-skilled employees is also the subject of a paper [Gérard 2016], in which the authors plan weekly tours of employee deployment. To find a solution, the authors developed 4 methods based on the column generation method. The solution designed by them is already applied and used in the planning software of the selected trading company. An interesting problem concerning the determination of the optimal number of employees is addressed by the authors of the article [Walter 2016] who in their study focus on the area of project management and with the help of linear programming try to minimize the average size of project teams in the conditions of simultaneous solution of multiple projects and with the possibility of involving one employee in multiple projects.

That being said, workforce optimization is an issue in almost every industry, and aviation organisations are no exception. A large number of papers focus mainly on optimization and planning of aircrews, such as the paper [Graf 2021]. Less attention has been paid by the authors to human resource planning in aircraft maintenance.

One of the first published integer mathematical models dealing with the minimization of maintenance staffing was in another paper [Yang 2003]. In the calculation, they work with three types of flexibility - flexibility in the deployment of individual shifts, flexibility in the composition of individual shifts and flexibility in working hours. The computational experiments were carried out on the actual maintenance schedule of a major Taiwanese carrier. Furthermore, the authors of the article [Permatasari 2019] present in their article a simple linear mathematical model where the optimization objective is to minimize personnel costs and the basis of minimizing the number of workers. The results can also be used to determine which activities and shifts a particular worker is assigned to. The authors consider a situation where maintenance staff have type licenses for multiple aircraft types. Heavy maintenance scheduling is also the focus of the paper [Deng 2021], which aims to develop a decision support tool for maintenance scheduling staff. Also, in the article [Pereira 2021] the authors describe the development of decision support software for aircraft maintenance planning. The software is based on an integer linear model. In this paper, the authors present a method to simplify the challenging computational process of the problem. In aircraft maintenance, it is also possible to encounter activities with a duration which exceeds the length of the shift, leading to overtime. This can result in reduced safety levels due to maintenance technician fatigue. The authors of the article [Xue 2022] focus on this issue. They present an approach that would eliminate the creation of overtime while ensuring the integrity of such activity with appropriate handovers between shifts. The optimization criterion is the total maintenance time, with the aim of minimizing it. In contrast to previous articles that focus on optimizing the number of maintenance technicians in heavy aircraft maintenance, this paper [Tang 2019] focuses on optimizing maintenance technicians in aircraft line maintenance.

This paper is inspired by the general model for human resource planning published in [Teichmann 2016]. This paper shows a modified mathematical model so that it can be used in heavy aircraft maintenance conditions, specifically to optimize the number of maintenance technicians needed for selected types of maintenance operations.

3 PROBLEM FORMULATION

The set of activities N forming the process is given. Each activity N is assigned a possible work specialization from the set of worker specializations s, denoted as L_S. The set of workers L is composed of subsets of $L_S \in L$, which includes workers of a given specialization $s \in S_i$. For each activity $i \in N$ is defined the set of specializations $S_i \in S$. For each specialisation $s \in S$ a constant Q_s is defined, which indicates the available number of workers for that specialization. Competence of specialization $s \in S$ to perform the activity $i \in N$ is modelled by the incidence matrix $\boldsymbol{A}$. If activity $i \in N$ can be performed by a worker of specialization $s \in S$, then a_{is}=1, if the activity cannot be performed by the worker, then a_{is}=0. The relevant time penalty for the activity $i \in N$ is modelled by the penalization matrix $\boldsymbol{B}$. In the case that activity $i \in N$ is performed by a worker of the assigned specialization $s \in S$, then b_{is}=0, if activity $i \in N$ is performed by a worker of the permitted specialization, then b_{is} takes the value of the corresponding time penalty, otherwise it takes the value of the prohibitive constant $b_{is}=W$. For each activity $i \in N$ is defined by its earliest possible start $t0_i$, latest permissible start $t1_i$, activity duration T_i, latest permissible end of activity Tk_i, and the number of workers g_i of specialization

$s \in S_i$. The optimization criterion is the number of deployed workers. The aim of the optimisation is to minimise the number of these employees. The solution of the Binary Programming Problem with Linear Bonds is also used to decide on the transfer of workers with different qualifications between activities.

Table 1 contains a summary of the symbols used in the Binary Programming Problem with Linear Bonds in the following section.

Symbol	Meaning
A	incidence matrix
a_{js}	element of the incidence matrix
B	penalty matrix
b_{is}	element of the penalty matrix
g_j	the number of workers to be assigned to carry out activity j
i	activity from the set N
j	activity from the set N
L_s	the set of workers of the specialization s
l_s	a worker from the set of specialization s
M	prohibitive constant
N	set of activities
Q_s	available number of workers with specialization s
s	worker specialization
S_i	the set of specializations carrying out an activity i
$t0_i$	earliest permissible start of activity i
$t1_i$	the latest permissible start of the activity i
T_i	duration of the activity i
Tk_i	the latest permissible end of the activity j
x_{ijsl}	bivalent variable representing the worker transfer
y_{js}	bivalent variable representing the assignment of the worker's specialisation s to activity j
z_i	non-negative variable representing the possible time delay of the start of the activity i
W	prohibitive constant

Table 1. Summary of notation used in the model

4 MATHEMATICAL MODEL

Objective function:

$$\min f(x,y,z) = \sum_{j\in N}\sum_{s\in S_j}\sum_{l\in L_s} x_{0jsl} \qquad (1)$$

Under the conditions:

$$\sum_{i\in N}\sum_{s\in S_j}\sum_{l\in L_s} a_{js}x_{ijsl} = g_j \qquad for\ j\in N \qquad (2)$$

$$\sum_{j\in N}\sum_{l\in L_s} x_{0jsl} \le Q_s \qquad for\ s\in S \qquad (3)$$

$$x_{ijsl} \le a_{js} \qquad for\ i\in N\cup\{0\},\ j\in N, s\in S_i\cap S_j,\ l\in L_s \qquad (4)$$

$$\sum_{j\in N} x_{ijsl} \le 1 \qquad for\ i\in N,\ s\in S_i\cap S_j, l\in L_s \qquad (5)$$

$$x_{ijsl} \le y_{js} \qquad for\ i\in N\cup\{0\},\ j\in N, s\in S_i\cap S_j,\ l\in L_s \qquad (6)$$

$$\sum_{s\in S} y_{js} \le 1 \qquad for\ j\in N \qquad (7)$$

$$\sum_{i\in N\cup\{0\}} x_{ijsl} = \sum_{i\in N} x_{jisl} \qquad for\ j\in N\ s\in S_i\cap S_j, l\in L_s \qquad (8)$$

$$t_i^{(0)} + z_i + T_i + b_{is} \le t_j^{(0)} + z_j + M(1-x_{ijsl}) \qquad for\ i\in N, j\in N,\ s\in S_i\cap S_j, l\in L_s \qquad (9)$$

$$t_j^{(1)} + z_j + T_j + b_{js} \le Tk_j + M(1-x_{ijsl}) \qquad for\ i\in N, j\in N,\ s\in S_i\cap S_j, l\in L_s \qquad (10)$$

$$z_i \le t_i^{(1)} - t_i^{(0)} \qquad for\ i\in N \qquad (11)$$

$$z_i \ge 0 \qquad for\ i\in N \qquad (12)$$

$$x_{ijsl} \in \{0;1\} \qquad for\ i\in N, j\in N,\ s\in S_i\cap S_j, l\in L_s \qquad (13)$$

$$y_{js} \in \{0;1\} \qquad for\ j\in N, s\in S_j \qquad (14)$$

The objective function (1) represents the total number of workers deployed.
A set of constraints (2) ensures that the required number of workers l are allocated to each activity. A set of constraints (3) will ensure that only the available number of workers are deployed. A set of constraints (4) ensures the assignment of the permissible specialization s to the activity j. The set of constraints (5) ensures that each worker l who can perform the activities i and j, either terminates its activity or switches to the activity j. The set of constraints (6) records the use of specialization in activity j. The set of constraints (7) shall ensure that the activity j is applied to at most one specialization s. Constraint group (8) ensures continuity of activities. The succession of activities guarantees the transfer of the worker from the previous activity to the upcoming activity. The set of constraints (9) will ensure the temporal continuity of the activities. The set of constraints (10) shall ensure that the activity is completed before its latest permissible end. The set of constraints (11) expresses the time interval in which the time delay of the start of the activity can be realised. The set of constraints (12) determine the definitional domain of the variables z_i used in the model. The values of the variables z_i can take real non-negative values. The sets of constraints (13) determine the definitional domain of the variables x_{ijsl} used in the model. The values of the variables x_{ijsl} can take bivalent values of 0 and 1. The sets of constraints (14) determine the definitional domain of the variables y_{js} used in the model. The values of the y_{js} variables can take bivalent values.

5 EXPERIMENTS AND RESULTS

The functionality of the Binary Programming Problem with Linear Bonds was verified by performing an experimental calculation in the field of aircraft maintenance, repair and overhaul organisation. The experimental calculation was performed in the Xpress-IVE optimization software.

This experiment used data from a real-life aircraft maintenance, repair and overhaul organisation. The data consisted of activities that are performed during initial heavy maintenance tests of a transport aircraft. This is one of the initial phases in the course of all heavy maintenance. Initial tests include functional and operational test activities designed to verify the proper operation and readiness of individual devices or entire aircraft systems. The experiment included activities that are mandated by the aircraft manufacturer based on the maintenance planning document as well as activities of a technological nature. Technological activities bring the aircraft to the desired state during maintenance so that the exact sequence of individual tests is followed. The activities were arranged in a schedule according to the individual parts of the aircraft so as to avoid collisions of the workers and thus their mutual influence in performance. The order of performance of each activity in a specific area of the aircraft was determined by its parameters and the probability of occurrence of findings from these activities. The continuity of technological activities was also essential so that their occurrence during maintenance was minimal. This process produced a schedule in which the critical path could be found using the CPM method. Specifically, there were 15 activities, with 9 activities being critical and 6 activities being independent. The experiment time was set at 7.14h due to the length of the critical path.

At that phase, 5 groups of workers, divided according to specialization, were allowed to be present. The worker's specialization was derived by the part of the aircraft to which he or she is primarily assigned. Cross-representation of specializations was allowed within the relevant qualification group. The first group of qualifications consisted of 2 groups of specializations, Avionics and Electronics. The second group of qualifications consisted of 3 groups of specializations, Interior, Airframe and Engines. Workers could be assigned to technological activities arbitrarily. Only a specialist from the relevant qualification group could be assigned to test activities. For each test activity, the worker's specialization was recommended relative to the part of the aircraft or its system in which the activity was performed. The time penalty for not assigning the recommended specialisation to an activity was set at 0.5 times the total duration of the activity. The time penalty is added to the total duration of the activity. The time penalty represents the time required to become familiar with a procedure that the worker does not normally perform. Assignment of a worker to a specialization other than the recommended one was only allowed for independent activities to maintain critical path length. In the case of a request to assign multiple workers to an activity, the Binary Programming Problem with Linear Bonds allows only a homogeneous group of workers to be assigned. In the experiment, 5 different specializations of workers were used according to each aircraft system.

Table 2 summarises the staff groups and their recommended availability in the heavy maintenance test phase.

Qualifications	Specialization	Availability
B1	Avionics	1
B1	Electronics	2
B2	Interior	3
B2	Airframe	2
B2	Engines	1

Table 2. Groups of available workers

Table 3 contains the results of the calculation.

Qualifications	Specialization	Calculation
B1	Avionics	0
B1	Electronics	2
B2	Interior	3
B2	Airframe	1
B2	Engines	0

Table 3. Groups of calculated workers

The optimization calculation of the variant was performed, and the optimal solution was found in the required computational time.

The result of the optimization calculation was a staffing requirement of 6 workers. According to the optimization calculation, 2 electronics workers, 3 interior workers and 1 airframe worker were required. The Avionics worker was not used in the calculation because he was represented by an Electronics worker for the activity where it was recommended. In this case, the duration of this activity has been extended. The same was the case for the Engine and Airframe specializations, where the worker was replaced by a worker of interior specialization who was available at the time but was used at another point in the experiment. This computational experiment has outlined the possibility of making a saving of 3 workers given the recommended availability on this part of the maintenance, with an appropriate sequencing of activities and given the assignment of workers. The recommended specialization was not assigned for 2 activities.

6 CONCLUSION

The article presented a solution to human resources optimization in processes that have different requirements for specialization of workers. A Binary Programming Problem with Linear Bonds assigning workers to activities was used to optimize the number of workers included in the work process.

The presented Binary Programming Problem with Linear Bonds will help optimize the number of employees needed to perform a selected process with different requirements for specialization of workers for individual activities. The condition of use is that the set of workers must be composed of different groups of workers with a different specialization with the possibility of substitution and must be allowed to shift the beginning of independent process activities.

The article introduced an experiment in the field of heavy maintenance of the aircraft. The calculation revealed a possible reduction in the number of workers and changes in the number of workers of the individual groups needed to perform initial heavy maintenance tests.

The number of workers deployed in the process is decisive in calculating the final product costs offered by the company on the market. The benefits of using this Binary Programming Problem with Linear Bonds published in the article is to reduce the total costs spent on the process, reduce the staff requirements of the process and increase the efficiency of the deployed workers. Other benefits include raising the awareness of workers about the principles of processes, as they will focus on one ongoing process and a reduction in workload, which occurs with unproductive time spent on orientation thought all ongoing processes in the whole work area. The task was to minimize the number of deployed workers under given conditions and requirements of activities.

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