ANALYSIS AND SIMULATION OF NONLINEAR SELF-EXCITED VIBRATIONS IN MACHINE TOOLS

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Commonly applied theories of self-excited vibrations in machine tools are prevailingly based on frequency domain analysis of linear models, and in most cases only focus on evaluation of stability limits of strictly linear systems. In this way the behaviour of more stable or more unstable systems cannot be studied and analyzed. Neither is it possible to study certain important nonlinear effects because of limitations connected with the classical representation of systems only in the frequency domain. Evaluation of stability by analysing nonlinear systems in the time domain has not been applied until recently. The paper shows a new approach based on an interpretation of self-excited vibration systems as nonlinear servo-systems. With this approach linear and nonlinear systems of any degree of stability can be studied using a combination of frequency and time-domain methods. This can considerably contribute to better understanding of complex phenomena in various regimes and specific situations.

Keywords
machining, machining process, machine tools, vibrations

1. Introduction

Self-excited vibrations in machine tools arise as a result of interaction between the elastic structure of the machine and the metal cutting process. Under certain conditions, typically at certain limiting chip widths, the interaction becomes too strong and the tool starts to vibrate relatively to the workpiece. The whole combined system becomes unstable; the amplitudes of vibration grow and may cause serious damage to the tool, spindle or the whole machine. Analyses of this phenomenon performed already fifty years ago by researchers of the VUOSO Research Institute of Machine Tools and Machining, namely [Tlusty 1954], [Tlusty 1955] and [Danek 1962], showed that the stability of excited vibrations is influenced by the space orientation of the machine vibration system into the normal of the machined surface. They also showed that chatter occurs at smaller chip widths when the machined surface already has certain surface waviness produced by the previous cut. Later [Tobias 1995] published the "lobe" diagrams, expressing how a change in spindle revolutions can influence the critical width of cut at the limit of stability. More recent publications generally applied the main achievements of the classical theory, based on the following assumptions:

- System stability can only be influenced by relative vibrations between the tool and the workpiece in the normal direction with respect to the machined surface
- A linear system is at the limit of stability when the amplitude of the vibrations Y(t) equals the amplitude of the surface waviness Y_0 (t) caused by the previous cut
- Since for constant values of the width of cut b the exciting cutting force F acting on the vibration system is considered as proportional to the current depth of cut (Y(t) – Y_0 (t)) by the specific cutting coefficient R, the depth of cut is always in phase with the cutting force F.
- As no phase shift, time delay or complex value of the coefficient R is considered, it follows that the real parts Re [Y(t)] and Re [Y_0(t)] will be equal in size and opposite in sense.

For a particular vibration system with the frequency response Φ, values of critical width of cut b_{lim} can be calculated relatively easily using the well-known criterion:

$$b_{lim} = -1/(2R \cdot Re[\Phi]_{neg} \qquad (1)$$

- These assumptions are valid for linear systems at the limit of stability and are commonly applied in most of the numerous later publications on the subject, such as [4], [6] and [7]. Nevertheless, to evaluate stability through the application of criterion (1), the method uses only the real part of the frequency characteristics of the machine structure as shown in the Fig. 1, and cannot be applied for a general analysis of system behaviour in both the frequency and the time domains.

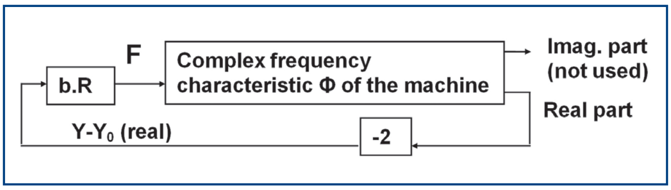

Figure 1. *Classical representation of the system for evaluating stability by applying criterion (1)*

2. Interpretation of self-excited vibrations as nonlinear feedback systems

For a more general analysis of the system's behaviour in both the frequency and the time domains, we have interpreted the waviness Y_0(t) as the vibration Y(t) delayed in time. For turning processes this time delay corresponds to the time T of one spindle revolution; for milling processes it corresponds to the time interval T/n between two cutting edges of the rotating tool. Modern Matlab/Simuling methods allow us to apply this nonlinear feature in both the frequency and the time domains. Then it is possible to introduce other nonlinearities, generate block diagrams of feedback systems for various cases and configurations, and study the behaviour of systems with different degrees of stability. Instead of only evaluating the chip width b_{lim} at the limit of stability, we may for instance study complete complex open loop characteristics and apply the Nyquist criterion of stability, applicable for linear and quasi-linear systems. The correspondence between the classical criterion (1) and the new Nyquist diagram is depicted in Figure 2. The Nyquist critical point for the limit of stability coincides with criterion (1), but the frequency response now includes the whole system, not only the mechanical structure.

The corresponding general block diagram for turning processes is shown in Fig.3. The depth of cut (Y(t)– Y_0 (t)) is represented here in a full complex form, not only as the parts Re [Y_0 (t)] and Re [Y(t)]. The programmed depth of cut H(t) represents the input function of the system. Coloured areas represent nonlinear blocks.

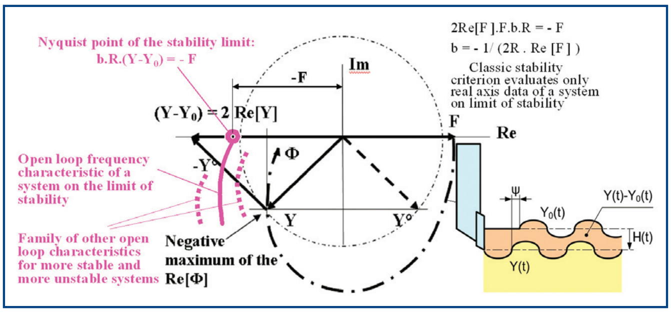

Figure 2. *Correspondence between the classic and Nyquist stability criterions*

The mean value of the input H(t) creates a particular mean value of the cutting force and a corresponding medium displacement of the machine structure, which does not theoretically influence the stability of a linear system. The current depth of cut h(t) is expressed by the equation:

$$h(t) = H(t) - (Y(t) - Y_0(t)) \quad (2)$$

When the value of (Y_0 (t) – Y(t)) is greater than the nominal depth of cut H(t), the tool jumps off the material and the cutting force F(t) stops being generated. In the diagram in Figure 3 this is represented by a nonlinear block which lets only positive h(t) values pass unchanged and blocks the negative ones. The tool jumping off the material is a nonlinear effect limiting the amplitude of vibrations in unstable systems. The positive part of h(t) represents the current depth of cut. Multiplication by the width of cut b produces a cross section of the chip; subsequent multiplication by the specific cutting resistance R produces the cutting force F(t). The classical stability criterion in equation (1) supposes that the R value is real, but we may also deal with a certain time delay Z between the chip cross section b·(Y(t) – Y_0(t)) and the cutting force F. Then, the depth of cut (Y_0(t) – Y(t)) will be represented in a full complex form, not only as the difference between the real parts Re [Y_0 (t)] and Re [Y(t)]. The constant value of the input H(t) creates a particular medium value of the cutting force and a corresponding medium displacement of the machine structure, which does not influence stability.

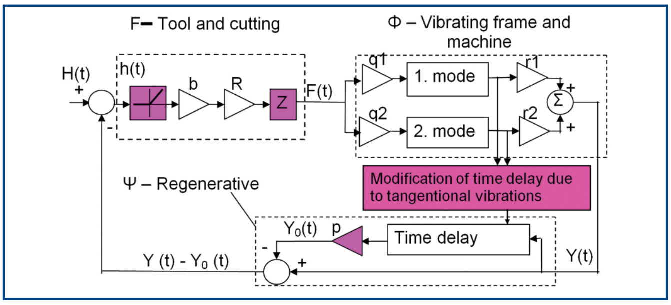

Figure 3. *General block diagram of a non-linear two-mode system*

When the value of (Y(t) – Y_0 (t)) is greater than the value of H(t), the tool jumps off the material and the cutting force F(t) stops being generated. This is represented in Figure 3 by a nonlinear block which lets only positive values of h(t) pass unchanged and blocks the negative ones. The tool jumping off the material is a nonlinear effect limiting the amplitude of vibrations in unstable systems. The positive part of h(t) represents the current depth of cut. Multiplication by the width of cut b produces a cross section of the chip; multiplication by the specific cutting resistance R produces the cutting force F(t). In Figure 2 possible complexity of the specific cutting coefficient R is represented by the time delay Z.

The whole part of the diagram containing the input h(t) and output F(t), in which the force F(t) is generated, is referred to as the subsystem F. In addition to the subsystem F the block diagram in Figure 3 comprises two other subsystems: the subsystem Φ and the subsystem Ψ. The subsystem Φ represents a two-mode vibration structure composed of the machine and tool and includes direction factors $q_{1,2}$ and $r_{1,2}$. These factors transform vibrations into the normal direction of the machined surface.

Vibration in the tangential direction can be considered as a modification of the time delay T in the other block subsystem Ψ, which solves the waviness effects of previously machined surfaces and performs the conversion of the vibration Y(t) into the current depth of cut (Y(t)– Y_0(t)).

The parameter p varies between one and zero and respects the fact that in turning only a part of the machined surface may enter into a repeating turning cut. Figure 3 shows examples of frequency characteristics of subsystems Φ and Ψ for a two-mode vibration system with natural frequencies of 72 Hz and 120 Hz. Tangential vibrations are not considered here. Figure 5 shows the resultant open loop frequency response of the system with all three interconnected subsystems F, Φ and Ψ. Parameter values are b = 1. 10^{-3} m, R = 2.10^9 Pa, Z = 0 sec, T = 0,1 sec; mode frequencies are 72 and 120 Hz. When a slight time delay Z of 0.002 sec is considered, corresponding to a reasonable phase shift at the frequency at the limit of stability, characteristics from Fig. 5 turn into shapes shown in Figure 6. For the given b value the system is no more stable.

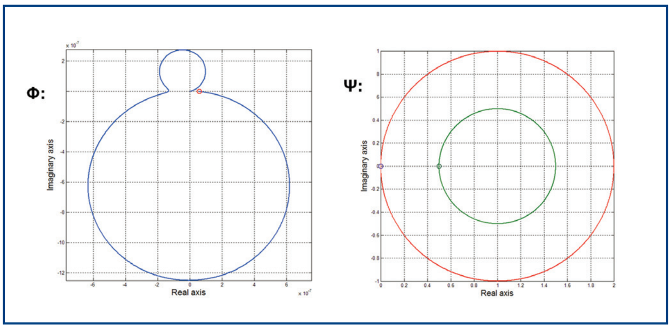

Figure 4. *Frequency response of the subsystems Φ and Ψ.*

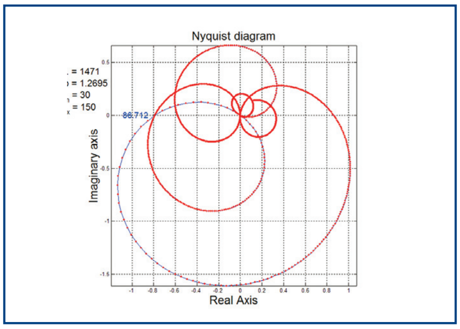

Figure 5. *Resultant open loop frequency response of the system with all three block subsystems F, Φ and Ψ interconnected.*

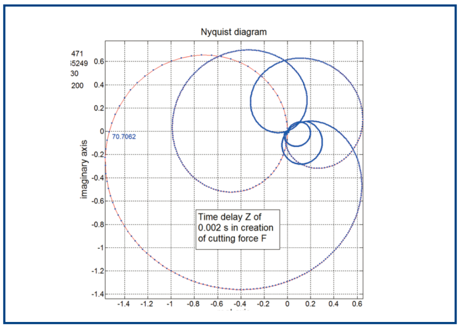

Figure 6. *Frequency characteristic modified due to Influence of time delay Z=0.002*

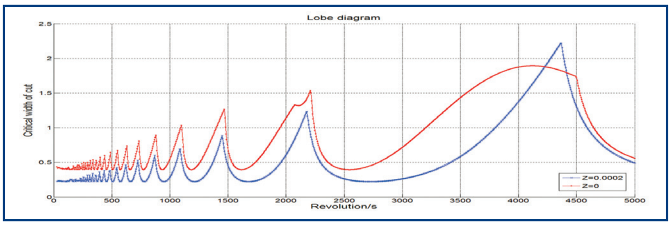

Figure 7. *Lobe characteristics for Z=0 and Z= 0.002 for a range of 30 to 5000 rev/min*

3. Special phenomena near the limit of stability

Changes in the spindle revolutions influence directly the values of the time delay T and the form of the open loop Nyquist characteristics shown in Figures 5 and 6. The application of the new procedures makes it possible to plot "lobe" diagrams showing the changes in critical b_{lim} for different spindle revolutions as shown in Figure 7 for both systems presented above. The system with the time delay Z shows significantly lower b_{lim} values in its "lobe" diagram.

Analysis has shown that the critical width of cut b_{lim} depends solely on the phase shift between the vibration Y(t) and surface waviness $Y_0(t)$. The corresponding graphical proof is shown in Figure 8; Figure 9 depicts a detail magnified 200 times. Self excited vibrations only occur in a certain phase shift range; in other areas they do not exist even with the most extreme b values. The situation is illustrated with 3D diagrams in Figures 10 and 11. The relation between the phase and $b_{lim.}$ remains unchanged; however, the occurrence of stability limits varies quite unexpectedly and jumps among different spindle revolutions.

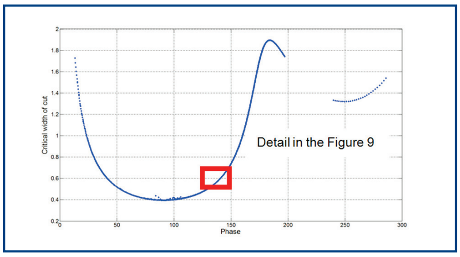

Figure 8. *At any spindle revolutions, the critical width of cut depends solely on the phase shift between the vibration and surface waviness*

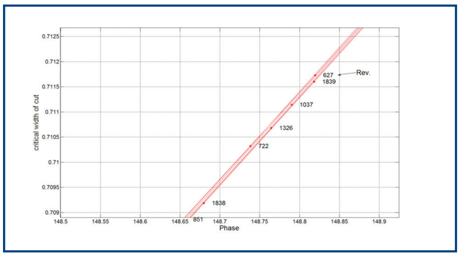

Figure 9. *A detail of a portion of Figure 8, magnified 200 times*

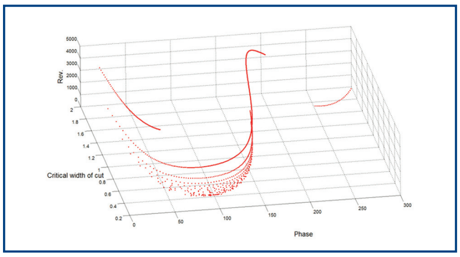

Figure 10. *A 3D representation of stability limits occurrence within a selected range of spindle revolutions*

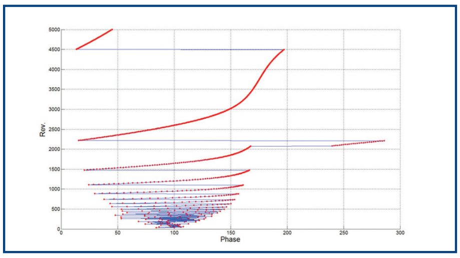

Figure 11. *Variations in phase and the corresponding critical width of cut when spindle revolutions change*

4. Time domain simulations

The criteria of stability used in the classical linear theory of self–excited vibrations is based on a comparison of two successive vibration amplitudes, these being the amplitude of current vibration Y and amplitude Y_0 of surface waviness.

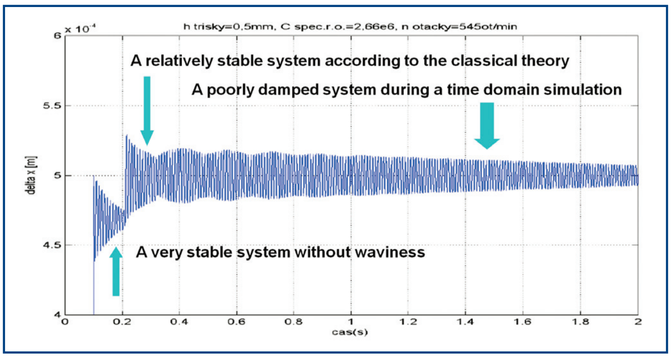

Figure 12. *Simulation in the time-domain showing effects of shocks from incoming surface waviness*

This approach supposes that the system at the limit of stability finds itself in a special state of steady vibrations, after subsiding all previous transient phenomena. The criterion may be referred to as a "steady-vibration" criterion. However, this presumption may not be valid in real situations. Vibration and surface waviness may interfere and combine with various process shocks. In this case a simulation in the time domain could provide a better understanding of the nature of the processes. The general block diagram shown in Fig. 2 can be used without modifications for this purpose, making it even possible to simulate the influence of specified nonlinear phenomena.

Figure 12 shows an example of a transient process simulation for a case where the tool starts to machine wave-less material. At the beginning the vibrations are well damped. After the time period T elapses, the system receives the first shock from incoming surface waviness. According to the classical "steady-vibration" criterion the system should be well damped; however, the time domain simulation only shows very poor damping due to periodically repeating shocks from incoming waviness.

5. Evaluating stability in time-domain simulation

Analysis and simulation of the transient phenomena of self-excited vibrations can also be applied to quantitative and evaluation of stability limit if a suitable stability criterion was found. Various criteria have been tested unsuccessfully, with cases of failure occurring each time. Finally an automatic iteration procedure called the "vibration

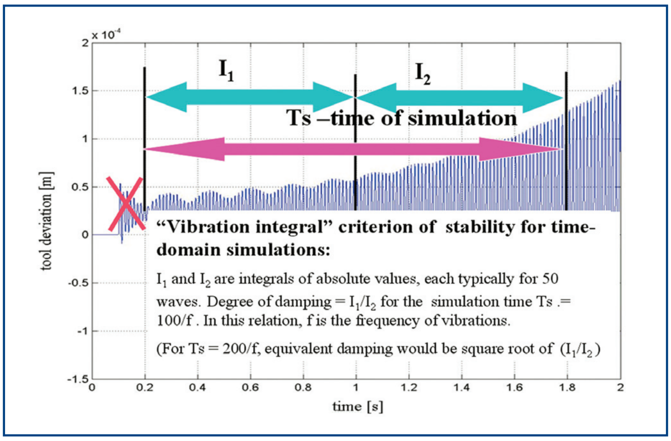

Figure 13. *„Vibration integral" criterion for quantitative evaluation of stability from time/domain simulation*

integral" criterion has been developed, which has so far been successful in all linear and nonlinear cases tested. The procedure is depicted in Figure 13 and involves the following steps. The transient process is simulated for a time longer than approximately 200 vibration waves. The first period, during which the tool cuts clean material without waviness, is not included in further analysis. The simulation time Ts is then divided into two equal time periods.

During the first period the integral of absolute vibration values is collected in one register where it creates the sum I_1; during the second period it is collected in another register where it creates the sum I_2. By iterative changes of the width of cut b we find the b_{lim} for which $I_1 = I_2$, and take it as the limit of stability. For other cases it is also possible to evaluate the ratio I_1/I_2 to obtain a quantitative evaluation of the degree of stability. The "vibration integral" criterion has worked well for all nonlinearities tested so far.

6. Lobe diagrams for systems with different degrees of stability

Being able to determine the limit of stability by evaluating b_{lim} obtained through simulated transient processes, we can repeat the same procedure for different spindle revolutions and draw the lobe diagrams for the simulated transient processes of linear or nonlinear systems. An example of such a lobe diagram is shown in Figure 14.

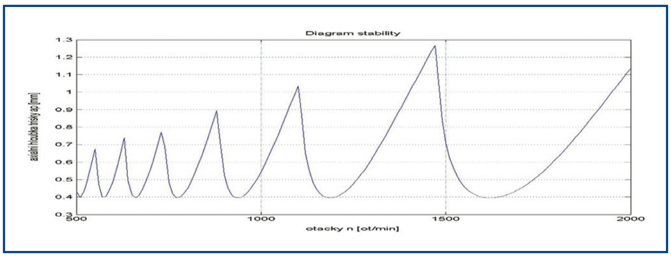

Figure 14. *Example of a lobe diagram obtained through iteration from time-domain simulation data*

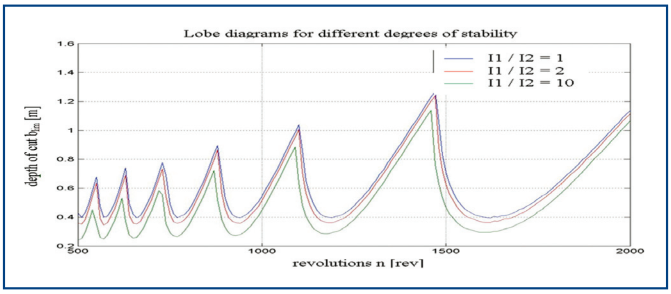

Figure 15. *Lobe diagrams for systems with different degrees of stability*

Moreover, with the quantitative "vibration integral" criterion, we can choose the desired degree of system stability by selecting the ratio (I_1/I_2), and draw the lobe diagrams for various "safety" against instability. Figure 15 shows three lobe curves for ratios (I_1/I_2) equal to 1, 2 and 10. Internal system parameters for systems with different degree of stability can be registered during the time-domain simulation.

7. Summary

A new approach to the analysis of self- excited vibrations involving non-linear elements has been applied in the Research Centre of Manufacturing Technology in Prague (RCMT). Interpretation of self-excited vibrations as non-linear feedback systems makes it possible to apply advanced Matlab/Simulink methods and study frequency and transient phenomena in systems with various degree of stability. Simulation and analysis of transient responses can contribute considerably to a better understanding of complex, nonlinear dynamic systems and of their behaviour in various regimes and specific situations.

List of symbols

Y	amplitude of tool vibration
Y_0	amplitude of surface waviness
H(t)	nominal depth of cut
h(t)	current depth of cut
F(t)	cutting force
R	specific cutting force coefficient
b_{lim}	width of cut on limit of stability
Z	time delay between chip cross section and F
T	time delay between successive cuts
T_S	time of simulation
I_1/I_2	degree of system stability

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