PARAMETRIC AND NONPARAMETRIC METHODS OF STATISTICAL PROCESS CONTROL

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This paper presents the limitations of classical Shewhart control charts and some possibilities of statistical process control that can be used when the basic assumptions about data have not been fulfilled. These basic assumptions that must be met include mainly a requirement on the normality of the data, the requirement for constant mean and variance, and last but not least the requirement for mutual independence of data. In practice, those assumptions about the data are not necessarily always met. The aim of this article is to introduce the problems (such as normality failure, data dependence) that can occur when applying the classic Shewhart control charts. Additional aim of this article is to describe some non-parametric control charts and concretely introduce one of the non-parametric control charts, namely Shewhart sign control chart, including a practical example from a metallurgical process. During preparation of this article accessible pieces of knowledge on the issue were compared. Comparing the parametric and nonparametric methods it was found that nonparametric methods have many advantages and for cases where some of the basic assumptions about the data are not met they are appropriate.

KEYWORDS
production management, statistical process control, Shewhart control charts, nonparametric control charts, Shewhart sign control chart

1 INTRODUCTION

Statistical process control (SPC) is an immediate and continuous process control based on the mathematical-statistical evaluation of the product quality. If a company wants to achieve the high quality consistently, it has to collect, process and analyze systematically data available from the production and conclusions of the analysis must be used for continuous improvement. To use the classic Shewhart control charts, the certain basic assumptions about the data must be met. In the manufacturing practice, however, it is not always possible to meet these basic assumptions. The aim of this paper is to emphasize the limitations of the classical Shewhart control charts and answer the question how to control the production process if not met the basic assumptions about the data.

2 CLASSICAL SHEWHART CONTROL CHARTS

Statistical process control allows interventions in the process based on the early detection of deviations from a predetermined level. The aim of the SPC is to keep the process at the required and stable level. It is implemented by regular monitoring of the controlled process variable or output variable. It is founded out whether it corresponds to the level required by the customer. Achieving the desired level of the process requires a thorough analysis of the process variability. [Nenadal 2008]

Most publications about the SPC deal with the processes that meet the basic requirements needed for the use of the classical Shewhart charts. These assumptions include: [Bakir 2001; Jarosova 2015]

- compliant capability of the measurement system
- normal distribution of the quality characteristics,
- constant mean and variance,
- mutual independence of quality characteristics values
- a sufficient quantity of data,
- sensitivity to greater changes in process,
- monitoring single quality characteristics per unit of product.

The main tool of statistical process control is the control chart. It is used to decide whether a process statistically stable or not. It shows the development of the process variability in time and uses principles of statistical hypothesis testing. Control chart consists of a central line *CL*, the upper control limit *UCL* and lower control limit *LCL*. Upper and lower control limits define the zone of influence of random causes of variability. On the *x-axis* the order of subgroups is plotted. On the *y-axis* the sample characteristic used as a test statistics in the control chart is plotted. The process can be considered as statistically stable if the values of the sample characteristic for all subgroups are within control limits and do not form any nonrandom pattern [Jarosova 2015]

2.1 The risk of false and missing signals

For the controlled variable (monitored quality characteristic or technological parameter) as a random variable, the hypothesis about the values of parameters of its probability distribution is being formulated. This null hypothesis should be formulated in the way that the process meets the quality requirements when this hypothesis is true (so that the process could be considered statistically stable). This null hypothesis is repetitively tested based on the regularly repeated, mostly small samples (rational subgroups). Rejecting the null hypothesis (points outside the control limits, trends or some non-random patterns) is the signal that the process with high probability deviated from the supposed state (it means that the process is out of control (it is not statistically stable) and some control action must be accepted and implemented. Control action equals to the identification and partial or total elimination of the assignable cause that caused the signalled undesirable changes in the process behaviour.

Null hypothesis H_0 in SPC means that the process is statistically stable; alternative hypothesis H_1 means that the process is not statistically stable. The area between control limits *LCL* and *UCL* in the control chart constitutes the domain of acceptance of the null hypothesis, and the area outside the control limits is the domain of rejection of the null hypothesis. Values of the control limits *LCL* and *UCL* are called critical values depending on the significance level α, i.e. the probability of the type I. error.

In SPC the probability of type I error (denoted as α) is called the risk of a false signal (false alarm). It represents probability of the vain search for an assignable cause based on the symptom of instability in the control chart (point outside the control limit or some non-random pattern) even if the process, in fact, has not changed (see Figure 1a)). This incorrect result is associated with the costs of searching for a non-existing problem.

In SPC the probability of type II. error (denoted as β) is called the risk of a missing signal. It is the probability that a control chart is not able to detect significant change of the process immediately after its appearance (there is no point outside the control limit, no instability pattern in a control chart). This incorrect result leads to the costs due to the missing control action. In Figure 1 b), c) this situation is depicted for a significant shift of the level of the controlled variable from the desirable target μ_0 to the undesirable (critical) level μ_1 or μ-1. The value (1 - β) is generally called the test power. It is the probability that the critical shift from μ_0 to μ_1 will be revealed in the first subgroup after its appearance. The graphical representation can be seen in Figure 1. [Jarosova 2015]

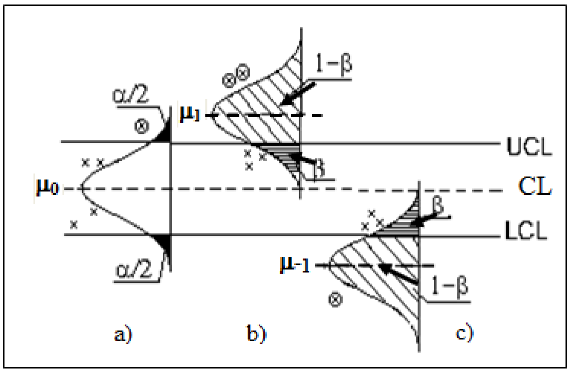

Figure 1. The risks of false alarm and missing signal [Jarosova 2015]

2.2 Data assumptions and their verification

Before selection and application of the classical Shewhart control charts assumptions about the distribution of the controlled variable must be checked. These assumptions include among others the independence of the data, a normal probability distribution and constant mean and variance. This verification has been performed using a variety of statistical tests or graphical tools. [Jarosova 2015]

2.2.1 Normality

Normal distribution is a prerequisite for the application of most statistical methods, including classical Shewhart charts.

If this assumption is not met, it is expected that the control chart will not have the expected properties. This also applies to other assumptions. For classical control chart a higher probability of false signals must be expected. For verification of normality there are many tests, which can be divided into directional or bidirectional tests and omnibus tests. The tests differ in their power, i.e. ability to detect various departures from normality. Shapiro-Wilk test and its modifications, Anderson-Darling test, R-test Jarque-Bera test are considered being the most powerful ones [D'Agostino 1986; Madansky 1988; Strelec 2010]. The modern approaches in the area of testing hypotheses about the normality of the data include robust tests [Strelec 2010]. The null hypothesis H_0 is defined for all normality tests as follows: assessed data come from a normal distribution. [Jarosova 2015]

In the next paragraphs the most powerful tests for normality checking are briefly described.

- Shapiro - Wilk test

  It belongs to omnibus tests, which are used in the case that there is no information about what kind of deviation from the normal distribution is to be expected. This test is based on the analysis of variance. The value of the test statistics reflects how close the points to the line of best fit lie. It is suitable for a range of choice $3 \le n \le 50$. [Jarosova 2015] The Shapiro-Wilk W statistic is basically the ratio of $\hat{\sigma}^2$ to S^2, the sample variance. In particular,

$$W = \frac{(a'X)^2}{(n-1)S^2} = \frac{(\Sigma a_i X_i)^2}{\Sigma (X - \bar{X})^2} \tag{1}$$

  where

$$a' = \frac{c'V^{-1}}{(c'V^{-2}c)^{\frac{1}{2}}} \tag{2}$$

  The a_i of (1) are the optimal weights for the weighted least squares estimator of σ given that the population is normal distribution. [D'Agostino 1986]

- Shapiro –Francia test

  Shapiro and Francia addressed the problem of *a* weights of the Shapiro – Wilk test by noting that for large samples the ordered observations may be treated as if they were independent. With this, the a weights of (2) can be replaced by

$$b' = \frac{c'}{(c'c)^{\frac{1}{2}}} \tag{3}$$

  and the *W* statistic of (1) can be replaced by

$$W = \frac{(b'X)^2}{(n-1)S^2} = \frac{(\Sigma b_i X_i)^2}{\Sigma (X - \bar{X})^2} \tag{4}$$

  c is the vector of the expected values of the *n* order statistics from the standard normal distribution. Values of *c* are readily available for *n* up to 400. [D'Agostino 1986]

- Royston modification

  Royston carried out further approximation of *a*, so extended interval for $7 \le n \le 2000$. [Madansky 1988]

- Anderson - Darling test

  It is a modification of the Kolmogorov - Smirnov test and it is used to identify the distribution of sample data. This test has specific critical values for each type of distribution. It belongs to a class of quadratic tests based on empirical distribution function (EDF). For the meaningful results of this test the selection of sample size $n \ge 50$ is recommended. [Kotlorz 2012; Salda 2010]

- Tests based on the skewness and kurtosis

One of the methods how to distinguish between two different distributions is to compare their central moments. For $k \geq 2$ is the central sample moment of kth order from a random sample $x_1, x_2, \ldots x_n$ defined by a relationship

$$m_k = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^k \tag{5}$$

For the skewness test the statistics below will be used

$$\sqrt{b_1} = \frac{m_3}{\sqrt{m_2^3}} \tag{6}$$

In the case of kurtosis the test statistics is determined by the relationship

$$b_2 = \frac{m_4}{m_2^2} \tag{7}$$

where H_0 is rejected when $\left|\sqrt{b_1}\right|$ it is larger than the critical value and b_2 does not lie in the interval between the upper and lower critical value. [Kotlorz 2012]

There are also tests that test both, kurtosis and skewness, simultaneously. One of them is Jarque - Bera test. For this test the sample of size $n \geq 200$ is recommended. The test statistic is as defined

$$JB = \frac{n}{6}\left[\frac{m_3^2}{m_2^3} + \frac{\left(m_4^2 - 3\right)^2}{4}\right] \tag{8}$$

the null hypothesis is rejected in case that the formula below is not true

$$\chi_2^2\left(1 - \alpha/2\right) \leq JB \leq \chi_2^2\left(\alpha/2\right)$$ [Kotlorz 2012]

- R-test

The simplest omnibus test consist of performing the $\sqrt{b_1}$ test at level α_1 and the b_2 test at level α_2 and reject normality if either test leads to rejection. The overall level of significance a for these two tests combined would then be, by Bonferroni´s inequality,

$$\alpha \leq \alpha_1 + \alpha_2 \tag{9}$$

Pearson, D´Agostino, and Bowman showed that if $\alpha_1 = \alpha_2 = 2\alpha^*$ a good approximation to the overall level of significance is

$$\alpha = 4\left(\alpha^* - \left(\alpha^*\right)^2\right) \tag{10}$$

The term R-test was given to the above omnibus procedure because it can be viewed as employing rectangular coordinates for rejection normality. [D'Agostino 1986]

- Q - Q chart

It is a graphical tool that enables to examine whether the data come from a certain type of distribution or not. The individual values are plotted on the *y-axis* and the quantiles $K_{\alpha j(X)}$ on the *x-axis*. Then, using the method of least squares the line of best fit is drawn through the points $[K_{\alpha j(X)}; x_{(j)}]$. The less the points from this line vary, the greater is the correspondence between theoretical and practical distribution. Example of Q-Q plot can be seen in Figure 2. [Salda 2010]

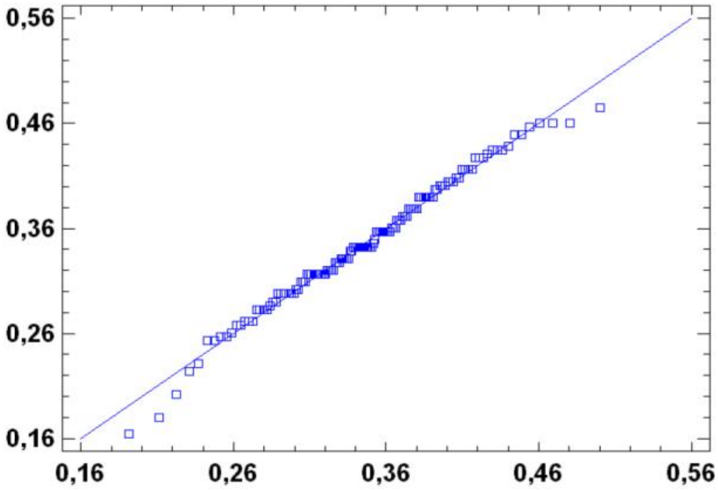

Figure 2. Q-Q chart [Salda 2010]

- Normal-probability plot

It is one of the P-P diagrams and can be used as an alternative to Q-Q graphs. The distribution function of selection is compared with a standardized distribution function of the selected theoretical distribution; in the case of normal probability plot with the distribution function of the normal distribution. [Jarosova 2015]

When deciding on the validity of the assumption of normality it is advisable to choose a combination of multiple tests. If the test results match the decision is easy. If the test results are different, it is necessary to carry out their thorough analysis and find the cause of these contradictions. [Jarosova 2015]

2.2.2 *Independence*

Independence of data can be expressed as a random variation around the mean value where no dependency appears. The null hypothesis is defined as H_0: data are independent. To verify independence the following tests may be used:

- Autocorrelation

The standard attack on the question of whether the x_i are independent is to check for serial correlation by correlating to series $\{x_i\}$ with the series $\{x_{i-l}\}$, where l is the size of the gap between observations being correlated. The usual resolution is to define the lth autocorrelation as [Madansky 1988]

$$r_l = \frac{\sum_{i=l+1}^{n} (x_i - \bar{x})(x_{i-l} - \bar{x}) / (n-l)}{\sum_{i=1}^{n} (x_i - \bar{x})^2 / n} \quad (11)$$

- Runs Above and Below the Median

Let v be the median of the distribution of x. Suppose we associate with each x_i a variable u_i, where

$$u_i = \begin{cases} 1 & \text{if} \quad x_i \geq v \\ 0 & \text{if} \quad x_i < v \end{cases}$$

Consider now the sequence $u_i,\dots,u_n$ and let p denote the number of u_i which are equal to 1. Conditional on the value of p there are $\binom{n}{p}$ possible sequences of p 1´s and n-p 0´s of which our observed set is one of these sequences.

One statistic based on the sequence of u_i which is a useful indicator of the independence of the u_i is the "runs count". We define a "run" as a maximal consecutive set of u_i´s having the same values. The sequence of u_i´s can be counted.

A low runs count is indicative of one kind of deviation from independence, namely a tendency for below - median x´s and above - median x´s to be observed in clusters. A high runs count is indicate of another kind of deviation from independence, namely a tendency for a below - median observation to be followed by an above - median observation. [Madansky 1988]

- Test iterations up and down

In this test, the "+" sign assigns a value if it is greater than the previous value x_i + 1> x_i and "-" sign in the opposite case, x_i + 1 <x_i. [Jarosova 2015]

2.2.3 *Homogeneity of means and variances*

Equality of the mean values can be tested using analysis of variance ANOVA. The null hypothesis and alternative hypothesis are defined as

H_0: $\mu = \mu_0$

H_1: $\mu \neq \mu_0$.

For verification of variance homogeneity there are various tests. But they differ in their robustness and power. The null hypothesis H0 and alternative hypothesis H1 for such tests are defined as follows:

H_0: $\sigma^2_1 = \sigma^2_2 = \dots = \sigma^2_M$.

H_1: At least one pair of variances differs [Jarosova 2015; Madansky 1988]

Before selection of the suitable variance homogeneity test it is necessary to verify if the samples are independent and if the data are normally distributed. In addition equality of samples in size must be taken into account, too.

In Table 1. there are summarized presumptions for application of five variance homogeneity tests. *"Y"* means that given test requires meeting the labelled presumption, *"N"* means that it is not necessary to meet the labelled presumption.

	Presumptions			
Test	Sample indepen-dence	Sample size equality	Number of samples	Normality of data in samples
Hartley	y	y	> 2	y
Cochran	y	y	> 2	y
Bartlett	y	N	> 2	y
Levene	y	N	> 2	N (robust to the smaller departures from normality)
Brown-Forsyth (modified Levene)	y	N	> 2	N (robust to the larger departures from normality)

Table 1. Variance homogeneity tests and presumptions for their application [own source]

Additional information on this tests can be found in [Howard 2010].

When the particular presumption for the application of classical Shewhart control charts has not been met it is possible to apply some of numerous non-classical parametric control charts defined for such situation. The basic summary of these control charts can be found in Table 2. [Jarosova 2015].

Situation	SPC method
Data non-normality	Control charts with asymmetric limits Control charts with re-transformed limits
Non-constancy of distribution parameters	Modified control charts Acceptance control charts Regression control charts Control charts with relaxed limits
Auto-correlated data	ARIMA charts Charts for EWMA residuals Dynamic EWMA charts
Low level of the process repeatability	Target control charts Standardized control charts Q-charts Hillier´s method
Small shifts in the process	CUSUM charts EWMA charts
Monitoring several characteristics simultaneously	Hotelling´s charts MCUSUM chart MEWMA chart
High-yield processes	CCC, CCC-r charts CCC CUSUM, CCC-r CUSUM charts CCC-EWMA, CCC-r EWMA charts

Table 1. Selected non-classical parametric control charts [Jarosova 2015]

Application of the parametric methods of statistical process control requires knowledge of the methods for verification of data presumptions and expert knowledge of various parametric control charts. As it is evident from the previous parts of the paper there are numerous methods for the data presumptions verification and the selection of the correct test also supposes knowledge and verification of the conditions in which the particular test is sufficiently powerful. Thus in spite of the SW support, the application of the parametric control charts is rather complex.

3 NONPARAMETRIC CONTROL CHARTS

In non-compliance with data assumptions for application of the classical Shewhart control charts (see chapter 2 of this paper), it is also possible to apply nonparametric methods. Nonparametric statistical process control (NSPC) is based on methods that are not dependent on a specific type of the probability distribution. The use of these control charts is not only suitable for processes that do not meet normality and independence of the data, but especially in the beginning of the SPC implementation, when there are not enough data available. [Chakraborti 2001]

Nonparametric methods are based on a smaller number of observations. Compared to the model based methods most often it is only assumed that the probability distribution of the given data set is of the continuous type. [Zvarova 2011]

Nonparametric methods have, compared to parametric methods, a number of advantages:

- conclusions obtained are independent of the distribution shape,
- they can be used even when the type of distribution is unknown,
- they are used in cases where sample size is too small,
- they can be used for ordinal (serial) variables, some also for nominal (verbal) variables,
- for small sample size the calculation is relatively simple,
- they have a greater robustness to the occurrence of outliers.

Disadvantages of non-parametric methods include the increased probability of missing signal, which means that it often leads to incorrect non-rejection of untrue null hypothesis. This probability can be reduced by increasing the sample size. [Chakraborti 2001, Stiglic 2009]

Below there are some nonparametric methods that can be used if the basic assumptions, such as data normality, mutual independence or constant mean and variance are not met.

- Shewhart Sign Control Chart

  It is one of the simplest non-parametric control charts. It is based on simple statistics that tracks the difference between the number of observations above and below a predetermined target value. [Bakir 2015, Chakraborti 2001]

- EWMA-DFCC (Exponentially-Weighted Moving Average – Distribution Free Control Chart)

  It is a nonparametric control chart of exponentially-weighted moving averages. It combines the properties of the classical EWMA chart with the robustness of nonparametric charts. Hackl and Ledolter [Hackl 1992] considered the use of nonparametric control chart for individual observations using a standardized series of the observations. The simulation studies showed that the method is resistant to the outliers and works well even with sudden changes in the process. [Bakir 2001, Graham 2011, Chakraborti 2001]

- CUSUM-DFCC (Cumulative Sum – Distribution Free Control Chart)

  CUSUM charts are suitable in the case of sequential nature of the process control. One of the problems of the Shewhart sign control chart is the fact that the target value must be known. Application of the nonparametric CUSUM control chart is one of the ways to avoid this problem. This technique was originally developed by McGilchrist and Woodyer [McGilchrist 1975] for monitoring the amount of rainfall. [Bakir 2001, Graham 2011, Chakraborti 2001]

- Pre-control

  Ledolter and Swersey [Ledolter 1997] considered the possibility of using the pre-control as one of the alternatives to the Classical control charts. When comparing the standard control charts to the pre-control, they found out that the pre-control is of some importance, especially in machining. However, in general the pre-control is not an adequate substitute for the control charts. [Chakraborti 2001]

- Change- point chart

  This is a problem, when the change of distribution type occurs in the number of independent random variables after the first observation. It is an issue dealt by Bhattacharya and Frierson [Bhattacharya 1981]. The aim is to detect an unknown change-point without a large number of false signals and without having to know any assumptions about the type of probability distribution data. The nonparametric control chart was designed based on the weighted sum of values in a row and on the asymptotic behavior of the cumulative sums, assuming that there are small changes in distribution after a large number of observations. [Chakraborti 2001]

- Bootstrap Method

  It is a relatively young method. It was not practically possible to use this method before the advent of computers [Hall 1992]. It is a resampling of the original data set. From the original data there are generated bootstrap samples from which there are calculated samples characteristics $\hat{\theta}$, it is repeated *k*-times. The values $\hat{\theta}$ obtained from bootstrap samples are used to calculate the characteristics θ of the original data set. [Chernik 2008]

4 SHEWHART SIGN CONTROL CHART

Let $X_{i1}, X_{i2}, \ldots, X_{in}$ (i=1, 2, ...) denote (i = 1, 2, ...) sample or subgroup of independent observations of size n > 1 from a process with an unknown continuous distribution function F . Let θ_0 denote the known or specified target value. Let's compare each x_{ij} (j = 1, 2, ..., n) with θ_0 . Let's record the difference between θ_0 and each x_{ij} ($x_{ij} - \theta_0$). There will be n such differences, for the i^{th} sample. Let n^+ denote the number of observations with values greater than θ_0 in the i^{th} sample. Let n^- denote the number of observations with values less than θ_0 in the i^{th} sample. Sum $n^+ + n^- = n$, if the number of zero signs is zero.

Shewhart sign statistic is defined as

$$SN_i = \sum_{j=1}^{n} sign\left(x_{ij} - \theta_0\right) \tag{12}$$

where

$sign\left(x_{ij} - \theta_0\right)$ = -1, 0 or +1, if $\left(x_{ij} - \theta_0\right)$ < 0, = 0 or > 0.

Then SN_i is the difference between n^+ and n^- in the i^{th} sample, i.e. SN_i is the difference between the number of observations with values greater than θ_0 and the number of observations with values less than θ_0 in the i^{th} sample. The control limits and the center line of the two-sided nonparametric Shewhart-type sign chart (for the median) are given by $UCL = c$, $CL = 0$ and $LCL = -c$.

$$c = 2 \cdot t - n \tag{13}$$

where c ϵ{1, 2, ..., n} and n is a subgroup size. Value t can be obtained from table for binomial distribution for θ =0,5 - see Gibbons and Chakraborti [Gibbons 2003].

When the control chart includes all points between the control limits, the process is in-control. If any point is located on one of the control limit, if it is below the lower control limit or above the upper control limit, it means that the process is out-of-control. In this case, we have to find the cause and implement corrective measures. [Graham 2008]

5 EXAMPLE

The following example illustrates the application of Sign Shewhart Control Chart on the data obtained from the steelmaking process (Table 3). There was measured carbon content in the steel each day. The measured values are recorded in the table, in the columns x_1 to x_5. The target value $\theta_0 = 1.29\%$. With this value we will compare the data in the thirty-one subgroups each of five units. For each value we will compute the difference between the measured and the target value $sign\left(x_{ij} - \theta_0\right)$ and we will write it into the table as -1, 0, or +1 depending on whether the released $sign\left(x_{ij} - \theta_0\right)$ < 0, = 0 or > 0. Subsequently, for each subgroup we will calculate the value SN_i according to the formula (12). Now we can proceed to the calculation of control limits, according to the formula (13) and construct the control chart (Figure 3.). Subgroup size is n = 5 and from table [Gibbons 2003] t = 5, so the value $c = 2 \cdot 5 - 5 = 5$. It follows that UCL = 5, CL = 0 and LCL = -5.

Datum	x_1	x_2	x_3	x_4	x_5	SN_i
1.1.2015	1.272	1.767	1.601	1.348	1.534	3
	-1.00	1.00	1.00	1.00	1.00	
2.1.2015	1.454	1.324	1.435	1.193	1.237	1
	1.00	1.00	1.00	-1.00	-1.00	
3.1.2015	1.509	1.886	1.546	0.882	1.787	3
	1.00	1.00	1.00	-1.00	1.00	
4.1.2015	1.038	1.349	0.898	1.393	1.671	1
	-1.00	1.00	-1.00	1.00	1.00	
5.1.2015	1.094	1.232	1.612	1.08	1.376	-1
	-1.00	-1.00	1.00	-1.00	1.00	
6.1.2015	1.361	1.53	1.083	1.167	1.325	1
	1.00	1.00	-1.00	-1.00	1.00	
7.1.2015	1.024	1.542	1.313	1.371	1.072	1
	-1.00	1.00	1.00	1.00	-1.00	
8.1.2015	0.933	1.394	1.13	1.335	1.376	1
	-1.00	1.00	-1.00	1.00	1.00	
9.1.2015	1.168	1.121	1.264	0.95	0.903	-5
	-1.00	-1.00	-1.00	-1.00	-1.00	
10.1.2015	1.182	1.062	1.016	1.345	1.064	-3
	-1.00	-1.00	-1.00	1.00	-1.00	
11.1.2015	1.528	1.571	1.501	1.242	1.217	1
	1.00	1.00	1.00	-1.00	-1.00	
12.1.2015	1.077	0.826	1.062	1.283	1.396	-3
	-1.00	-1.00	-1.00	-1.00	1.00	
13.1.2015	1.169	1.289	1.476	1.745	1.479	1
	-1.00	-1.00	1.00	1.00	1.00	
14.1.2015	1.373	1.194	1.001	1.082	0.909	-3
	1.00	-1.00	-1.00	-1.00	-1.00	
15.1.2015	1.214	1.198	1.413	1.49	1.137	-1
	-1.00	-1.00	1.00	1.00	-1.00	
16.1.2015	1.281	0.986	1.509	1.579	1.388	1
	-1.00	-1.00	1.00	1.00	1.00	
17.1.2015	1.281	1.214	1.391	1.327	1.051	-1
	-1.00	-1.00	1.00	1.00	-1.00	
18.1.2015	1.106	1.407	1.508	1.558	1.173	1
	-1.00	1.00	1.00	1.00	-1.00	
19.1.2015	1.295	0.99	1.636	1.383	1.507	3
	1.00	-1.00	1.00	1.00	1.00	
20.1.2015	0.912	1.271	1.633	1.378	1.483	1
	-1.00	-1.00	1.00	1.00	1.00	
21.1.2015	1.695	1.044	1.65	1.536	1.46	3
	1.00	-1.00	1.00	1.00	1.00	
22.1.2015	1.317	1.185	1.218	1.778	1.613	1
	1.00	-1.00	-1.00	1.00	1.00	
23.1.2015	1.507	1.244	1.158	0.941	0.855	-3
	1.00	-1.00	-1.00	-1.00	-1.00	
24.1.2015	0.716	1.029	1.248	1.046	0.757	-5
	-1.00	-1.00	-1.00	-1.00	-1.00	
25.1.2015	1.206	1.145	0.772	1.109	1.137	-5
	-1.00	-1.00	-1.00	-1.00	-1.00	
26.1.2015	1.213	1.549	1.463	1.468	1.713	3
	-1.00	1.00	1.00	1.00	1.00	
27.1.2015	1.276	1.223	1.32	1.199	1.321	-1
	-1.00	-1.00	1.00	-1.00	1.00	
28.1.2015	1.419	0.968	1.032	1.107	1.066	-3
	1.00	-1.00	-1.00	-1.00	-1.00	
29.1.2015	0.982	0.706	1.174	1.044	1.137	-4
	-1.00	-1.00	-1.00	-1.00	-1.00	
30.1.2015	1.067	1.309	1.046	1.031	0.989	-2
	-1.00	1.00	-1.00	-1.00	-1.00	
31.1.2015	1.169	0.909	1.637	1.038	1.04	-2
	-1.00	-1.00	1.00	-1.00	-1.00	

Figure 3. Data [own source]

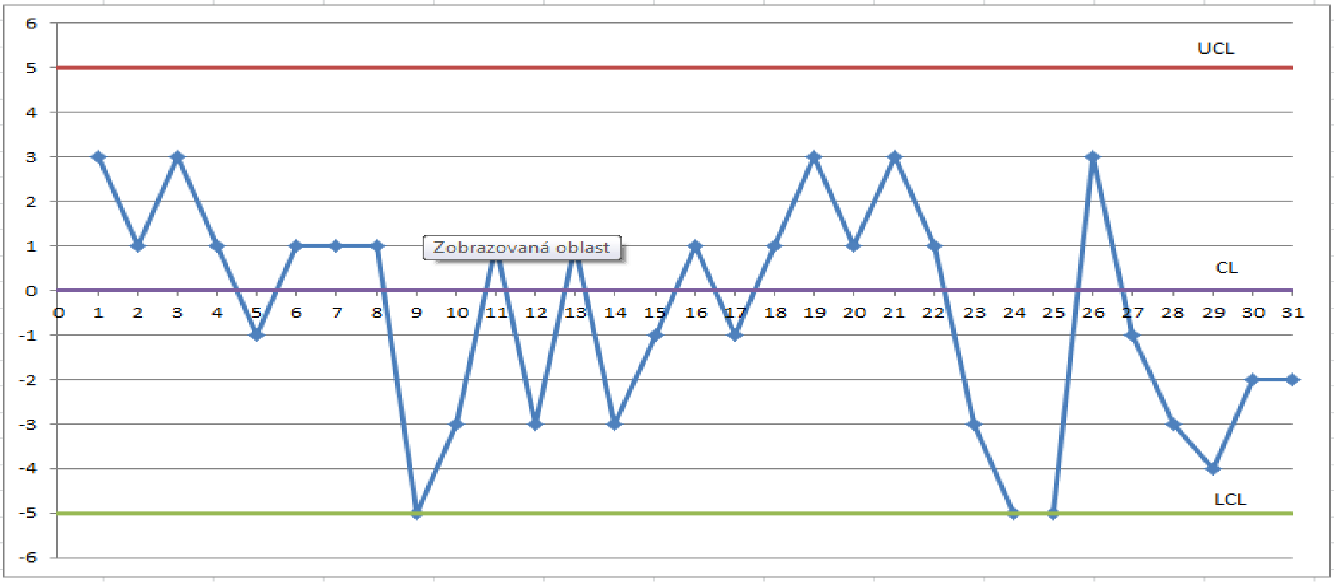

Figure 4. Shewhart sign control chart

The chart shows that in the ninth, twenty-fourth and twenty-fifth subgroup point lies on the lower control limit, which may mean exposure to assignable causes of variability that should be analyzed and subsequently eliminated. In the classical Shewhart control chart for average there did not appear the assignable cause in the process.

6 CONCLUSIONS

This article summarizes some of the shortcomings of classical Shewhart control charts (such as the necessity of normal distribution of data, mutual independence of data, and more). It offers the possibility of using non-parametric control charts, which eliminates these drawbacks. Specifically, it represents one of the non-parametric control chart and on the practical example illustrates the simplicity of its use in practice. The aim of the further work is a detailed look at how to control the production process, when some of the basic assumptions about the data are not met, and creating a methodology for control of such production process. The results will contribute to the development of statistical process control and process capability analysis. The proposed methodology could help in the decision-making processes in practice.

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