ELLIPSOIDAL DESIGN OF SLIDING MODE CONTROL FOR CAR ACTIVE SUSPENSION SYSTEM

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ABSTRACT

The issue of state feedback sliding mode control (SMC) for a particular kind of active quarter-car suspension system is discussed in this work. The suspension systems' dynamic system is built with the three control objectives of maximal actuator control force, suspension deflection, and ride comfort in mind. The next step is to build the state-feedback controller with the disturbance (vibration) attenuation level in mind to guarantee the asymptotic stability of the closed-loop system. The algorithm for SMC design is introduced. It is predicated on choosing the sliding surface correctly using the invariant ellipsoid approach. The system motion in the sliding mode is guaranteed to be little affected by mismatched disturbances according to the control architecture. Furthermore, the presence of admissible controllers is articulated in terms of LMIs through the use of Lyapunov theory and the linear matrix inequality (LMI) technique. The objective is to create a desired dynamic state-feedback controller when these requirements are met. Lastly, a quarter-car model is used to illustrate how successful a suggested approach is.

KEYWORDS

Active car suspension, state-feedback control, Sliding mode control, method of Invariant ellipsoid

1 INTRODUCTION

In order to give passengers a smooth ride, the suspension system, an essential part of the car, shields them from the vibrations caused by uneven road surfaces. Many efforts have been made in the last few years to study various vehicle suspension systems, including active, semi-active, and passive suspensions [1, 2]. These latter are capable of applying a force that modifies the suspension's properties and enhances driving comfort by adding or removing damping in a way that improves the suspension's ability to adjust to the terrain. Active suspensions have a greater ability to achieve strict performance requirements and demonstrate superior performance in enhancing ride comfort and safety when compared to semi-active and passive suspensions. Active car suspensions have gained more and more attention lately due to their benefits. Numerous scholars have dedicated their time to the theoretical advancements of active suspension design, employing a variety of control laws, including model predictive control [5], H∞ control [4], and linear quadratic regulation [3]. Numerous control systems, including preview control and multi-objective control, have also been documented [6]. Even if the theoretical findings appear promising, active vehicle suspension is not widely used in industry (partly because to its expensive cost). In academic research, simpler suspension models, like a linear quarter-car model, are frequently taken into consideration to facilitate theoretical development. In addition, actual systems invariably face non-ideal situations like as uncertainty, time delays, and non-linearities. To tackle this problem, various enhanced control techniques have been studied. One method for handling non-linearity and uncertainty is the linear parameter-varying technique (LPV). Ref [7] describes the LPV approach used for a quarter-car suspension system design with non-linear suspension components. However, quantifiable and bounded non-linearities are the major problems that the LPV theory can solve [8]. Feedback linearization is an additional option for non-linear control design that transforms non-linear system dynamics into completely or partially linear dynamics ; work by [8] demonstrates applications of this method in non-linear suspension design. Other techniques used in active non-linear suspension design are backstepping [9] and adaptive control [10]. In [11], a straightforward proportional derivative (PD) control with particle swarm optimization is provided. It is robust against changes in the load of passengers with suspension and damper nonlinearity. Disturbance rejection (attenuation) control methods are recommended because road abnormalities, which represent external disturbances, might affect the dynamics of the car and negatively impact the comfort of the passengers. Active disturbance rejection control (ADRC), regional pole placement, the invariant-ellipsoid method, fuzzy control, H∞ control, and sliding mode control are some of the frequently used disturbance rejection techniques. A more modern addition to the traditional proportional-integral-derivative (PID) control is the (ADRC). The ADRC has garnered a lot of attention recently since it was formally introduced by [12]. This approach uses an extended state observer (ESO) to estimate the entire disturbance in real time, which includes the external disturbance and unknown system dynamics. The control action then makes up the difference. While ADRC is simple to use and performs well, it is mostly independent of the system's mathematical model. Though uncommon, this method of designing active suspension has been embraced by [13, 14]. A robust state feedback control design is presented in Ref.[15] using linear matrix inequalities (LMI). This design ensures, in the presence of system uncertainties, that the closed loop system will satisfy the desired pole region, resulting in satisfactory oscillation damping and settling time, as well as guaranteed cost performance. Sufficient control in both discrete and continuous time with localized pole positioning while adhering to the automobile actuator force practical limit is provided in [16, 17]. [18] provides an invariant-set design for vehicle suspension control with actuator force restrictions. H∞ controllers are proposed in [4] for active suspension systems. Fuzzy control is another feature of the control system that finds widespread application in the industry due to its good robustness and simplicity of implementation. According to [19], fuzzy control has been adopted for active suspensions. Fuzzy control, on the other hand, relies on professional knowledge of appropriate membership functions and fuzzy rules. This endeavor becomes considerably more challenging in complex, non-linear systems. Designing an active suspension with a non-linear actuator typically involves the use of sliding-mode control (SMC)[20]. While sliding-mode control exhibits exceptional resilience against matched disturbances, its behavior may trigger undesired dynamic characteristics, potentially leading to a significant decline in performance in real-world scenarios. This phenomenon is known as chattering or extremely high-frequency oscillations that arise from the discontinuous nature of the control signals. The mitigation of this phenomenon through the use of discontinuous approximations [21,22]. Nevertheless, the robust performance is lost in such solutions. As we shall see later, the disturbance in the dynamics of the car is unparalleled, therefore a new sliding mode control is needed.

Thus, this study adds the following in light of the challenges in the introduction to stabilize the position of the sprung mass of the quarter car system:

• A simple design process that does not call for costly computational techniques.

• The SMC design process for vehicle active suspension systems is based on the methodology described in [23]. It is predicated on choosing the sliding surface correctly using the invariant ellipsoid approach. The control scheme ensures that mismatched disturbances and their influence on system movements in a sliding mode is minimized.

• The application of cutting-edge sliding-mode algorithms for quarter-car suspension system control.

• Using the features of Lyapunov functions, the closed-loop stability is ensured.

• The theoretical conclusions are validated by the simulation results.

1.1 Notations

Standard notations are used. The set of vectors of dimension n × 1 is called R^n. The real matrices with size n × m are called R^{nxm}. (.)′ indicates the transpose of a vector or matrix. A symmetric positive (negative) definite matrix is denoted by P > 0 (< 0). (*M + N+* ∗) is the notation for (*M + N +M′+N′*). Furthermore, the symbol (*) in a matrix denotes the symmetric portion, thus $\begin{bmatrix} M & N \\ * & S \end{bmatrix}$ means $\begin{bmatrix} M & N \\ N' & S \end{bmatrix}$. The symbols *0* and *I* stand for the zero matrix and the identity matrix, respectively. The time derivative of x is denoted by $\dot{x}$.

The order of the paper is as follows. The earlier research on automotive active suspension systems is presented in Section 1. The problem is formulated in Section 2. The ellipsoid approach of state feedback sliding mode control design is shown in Section 3. Results of a benchmark example simulation are provided in Sect. 4. Sect. 5 contains the conclusions.

2 SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Fig. 1 depicts a two-degree-of-freedom system using a quarter-suspension model that was taken from [4]. The sprung mass and the unsprung mass are represented in this model by the letters M and m, respectively. The vertical displacements x_s, x_u, and x_r, correspond to the sprung, unsprung mass, and road input, respectively. Moreover, the control force applied to the suspension is indicated by u. A collection of state variables

$$x_1 = x_s - x_u$$
$$x_2 = \dot{x}_s$$
$$x_3 = x_u - x_r$$
$$x_4 = \dot{x}_u$$

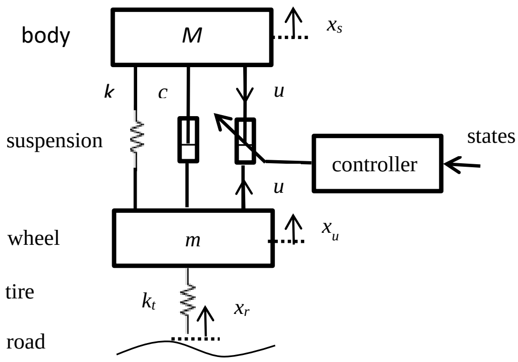

Figure 1. Two-DOF quarter-car model with active suspension.

The state equation [4] describes the dynamics of the model.

$$\dot{x} = Ax + Bu + Dw, x(0) = x_0 \quad (1)$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-k}{M} & \frac{-c}{M} & 0 & \frac{c}{M} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m} & \frac{c}{m} & \frac{-k_t}{m} & \frac{-c}{m} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{-1}{m} \end{bmatrix}$$

Here are the specifications of the so-called passive suspension (k, c) ; tire stiffness is denoted by k_t, and the sprung (body) and unsprung (wheel) masses are represented by *M* and *m*, respectively. In addition, is the suspension stroke, u is the scalar active force produced by a hydraulic actuator, x_r is the vertical ground displacement brought on by road unevenness, and, $x_u - x_r$ is the tire deflection. The definitions of the state variables in model (1) are,

$x_1 = x_s - x_u$, $x_2 = \dot{x}_s$, , $x_3 = x_u - x_r$ and $x_4 = \dot{x}_u$.

The active force is the control input, and the road roughness-related disturbance is represented by $\omega = \dot{x}_r$. Keep in mind that the external disturbance is *unmatched* (the matrix *D* does not lie in the range (*B*) i.e.

$D \neq B.\emptyset, \emptyset = any\ matrix)$.

Control design objectives include suspension performance such as road holding, suspension deflection, and ride comfort. It makes sense to select the acceleration of the body as the controlled output, *z(t)* as ride comfort may be measured by the acceleration of the body in the vertical direction. The resulting riding comfort increases with decreasing vertical body acceleration [18]. To enhance the comfort of the ride, one of the primary goals of the suggested controller is to reduce the vertical acceleration *z(t)* .[18] gives the body acceleration $\ddot{x}_s$.

$$\ddot{x}_s = z = C\ x + B_2 u \quad (2)$$

Where

$$C = \frac{1}{M}\begin{bmatrix} -k & -c & 0 & c \end{bmatrix}, B_2 = \frac{1}{M}$$

In addition, the suspension stroke should not go over the permitted maximum because of the mechanical construction. Alternatively, there should be a limit to the active control force that the active suspension system has available. The actuator's restricted power is represented by the expression $|u(t)| \leq u_{max}$, where u_{max} is the maximum actuator control force that may be applied. Put differently, actuator saturation limits the active forces produced by hydraulic actuators, which are regarded as control inputs.

Thus, the following state-space equations may be used to characterize the active vehicle suspension system based on the above conditions.

$$\dot{x} = Ax + Bu + Dw, x(0) = x_0$$

Subject to:

(i) the body acceleration $z = Cx + B_2u$ to be as small as possible (for satisfactory passenger comfort),

(ii) the external disturbance constraint

$$\|w(t)\| \leq 1, \ \forall t \geq 0 \quad (3)$$

(iii) the maximum actuator control force constraint

$$|u| \leq u_{\max} = \mu \quad (4)$$

Where *x(t), u(t), z(t), w(t)*, are the state, control input, output to optimize, and the external disturbance vectors of dimension *n, m, r, and p* respectively. The pairs *(A, B), (A, C)* are assumed controllable and observable respectively. Note that constraint (3) does not represent loss of generality. The external road disturbance, *w*, can always be normalized in the form (3) by proper selection of *D*.

The proposed state feedback control

$$u = Kx \quad (5)$$

has to stabilize the vehicle suspension system subject to the constraints (i-iii).

3 SMC BY THE INVARIANT ELLIPSOID METHOD

Consider the car suspension dynamics (1) put in a more general form

$$\dot{x} = Ax + Bu + Df \quad (6)$$

where $x \in R^n$ - the system state vector, $u \in R^m$ - the vector of the control inputs, rank(B) = m, disturbance vector $f \in R^k$. The system disturbance is assumed to be bounded

$$f'Qf < 1 \quad (7)$$

where the positive definite matrix Q is given.

Let the full state space vector x be measurable and system (6) is stabilized with the SMC of the form

$$u(t) = (-\alpha\|x(t)\| - \beta).sign[\tilde{C}x(t)] \quad (8)$$

where ||x|| is the Euclidean norm of the vector x, the control parameters α and β may be fixed. The matrix $\tilde{C} \in R^{m\times n}$ is the sliding surface to be found.

Note that car suspension dynamics is subject to unmatched road disturbance which cannot be rejected by the conventional SMC [21,22]

$$u(t) = -(\tilde{C}B)^{-1}\tilde{C}Ax(t) - M(x(t)).sign[\tilde{C}x(t)], \ M(x(t)) > 0$$

(9)

Here the matrix $\tilde{C} \in R^{m\times n}$ is a sliding surface such that det($\tilde{C}B$)$\neq$ 0 and the positive control gain function $M(x)$ has the form

$$M(x) = \sqrt{(\alpha + x'Rx)},$$

Where α,||R||≤β,α,β are scalars>0.

The primary challenge is creating a sliding mode control that uses the invariant ellipsoid approach to reduce (in a sense) the impacts of the unmatched disturbance [22, 23].

3.1 *Invariant ellipsoid method[23]*

Definition 1. The ellipsoid

$$E(P) = \{x \in R^n : x'P^{-1}x < 1\}, P > 0, \quad (10)$$

If any state trajectory of the system starting inside the ellipsoid stays inside it for all time instants $t > 0$, then the ellipsoid centered at the origin and a configuration matrix *P* are said to be state-invariant for the system (6) with respect to the disturbances (7); however, a trajectory starting outside the ellipsoid is attracted to this ellipsoid as time evolves (also termed attracting ellipsoid)[24].

One way to think about the invariant ellipsoid is as a feature of how unmatched perturbations (7) affect the system (6). The smallest (in a sense) invariant ellipsoid in the SMC (8) application offers a "minimum deviation of any possible trajectory from the origin in a sliding mode." The primary issue under consideration is creating a control of the kind (8) that will allow any system (6) trajectory to converge into the previously established "minimum invariant ellipsoid." For these kinds of optimization problems, the standard criterion is minimal *trace* (P^{-1}). A trace of the matrix P^{-1} describes the total of the squares of the ellipsoid's semi-axes.

An invariant ellipsoid for the linear disturbed control system was found in [24] using the technique of the (LMIs). Since choosing a suitable linear sliding surface $\tilde{C}x = 0$ makes up the majority of the SMC design, we may anticipate seeing comparable LMI forms in our case.

3.2 System decomposition and main result

Since rank (B) = m, the matrix B can be partitioned (maybe, after reordering the state vector components) as

$$B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

Where $B_1 \in \mathbb{R}^{(n-m)\times m}$, $B_2 \in \mathbb{R}^{m\times m}$ with det $(B_2) \neq 0$. In this case

The nonsingular coordinate transformation

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Gx\,, \quad where\ G = \begin{pmatrix} I_{n-m} & -B_1 B_2^{-1} \\ 0 & B_2^{-1} \end{pmatrix}$$

Reduces the system (6) to the regular form

$$\begin{cases} \dot{x}_1 = A_{11}x_1 + A_{12}x_2 + D_1 f \\ \dot{x}_2 = A_{21}x_1 + A_{22}x_2 + u(t) + D_2 f \end{cases} \quad (11)$$

where $x_1 \in \mathbb{R}^{(n-m)}$, $x_2 \in \mathbb{R}^m$ of the system state vector, of the system state vector

$A_{11} \in \mathbb{R}^{(n-m)\times(n-m)}$, $A_{12} \in \mathbb{R}^{(n-m)\times m}$, $A_{21} \in \mathbb{R}^{m\times(n-m)}$, $A_{22} \in \mathbb{R}^{m\times m}$, *are blocks of the system matrix*

$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = GAG^{-1}$ and $\begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = GD$

where $D_1 \in \mathbb{R}^{(n-m)\times k}$, $D_2 \in \mathbb{R}^{m\times k}$

Remark that if the system (1) is controllable, then the pair (A_{11}, A_{12}) is controllable too.

Theorem[23]: The solution $(\boldsymbol{\tau}, \boldsymbol{\delta}, \boldsymbol{Y}, \boldsymbol{X}, \boldsymbol{P})$ of the minimization problem

$$min\ tr(Z)$$

Subject to

$$\begin{bmatrix} (A_{11}X - A_{12}Y + *) + \tau X & D_1 \\ * & -\tau Q \end{bmatrix} < 0$$

$$\begin{bmatrix} \begin{bmatrix} X \\ X \\ -Y \end{bmatrix} & * \\ & GZG' \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} \begin{bmatrix} \delta X & 0 \\ * & L \end{bmatrix} & GD \\ * & \beta^2 Q \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} \begin{bmatrix} \delta X & 0 \\ * & L \end{bmatrix} & GA \\ * & \alpha^2 I_n \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} \frac{X}{\delta} & * & * \\ 0 & L & * \\ Y & L & I_m \end{bmatrix} \geq 0,$$

$$\tau \geq 0, X > 0, L > 0, \delta > 0$$

provides the sub-optimal invariant ellipsoid $E(Z^{-1})$ for the system (6) with the SMC (8), where the sliding surface is

$$\tilde{C} = (YX^{-1}, I_m)G$$

The above sliding surface is used in the proposed SMC

$$u(t) = -(\tilde{C}B)^{-1}\tilde{C}Ax(t) - M(x(t)).sign[\tilde{C}x(t)], \qquad M(x(t)) > 0$$

4 BENCHMARK EXAMPLE AND SIMULATION

An example is provided in this section to demonstrate the efficacy of the proposed design. The nominal values and system parameters are shown in Tab 1.

Parameter	Value
M	320 kg
m	40 kg
k	18 kN/m
kt	200 kN/m
c	1 kN.s/m

Table 1. Quarter-car active suspension parameters

The active force is bounded by u_{max}=allowable spring stroke (±0.08 m) × spring constant→1.5 kN. The state equation for the nominal plant is

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -56.3 & -3.1 & 0 & 3.1 \\ 0 & 0 & 0 & 1 \\ 450 & 25 & -5000 & -25 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 4.687 \\ 0 \\ -37.5 \end{bmatrix}$$

Since the invarianr-set control in [18] outperforms the H∞ and regional pole placer , we limit the comparison of the proposed SMC with the invariant-set design, and the conventional SMC(9).

Two test road profiles are presented : the random road , and the bump road profiles.

A. Random road response

Consider the random road test disturbance as vibration given by [18].

$$x_r(t) = 0.0254\, sin\, 2\,\pi t + 0.005\, sin\, 1\, 0.5\pi t + 0.001\, sin\, 2\, 1.5\pi t$$
(m) (12)

The Q matrix in (7) is evaluated from (12) (neglecting the cross product terms) as follows

$$Q_{1x1} = 1480 < \frac{1}{0.0254^2 + 0.005^2 + 0.001^2}$$

The proposed , conventional , and invariant-set controls are summarized in Tab2

Proposed SMC	**[0.391, 0.96322, -40.61, 0.1875]**	**Sliding surface**
Conventional SMC	**[278.47, 11.773, 8.0217, 14.45]**	**Sliding surface**
Invariant-set control[18]	***K*=[2.9044 , -1.9839,-3.4093, -0.32433]**	$u = Kx$

Table 2. Data for the proposed, conventional, and invariant-invariant controls.

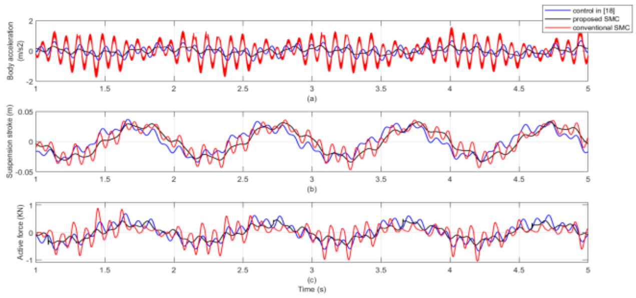

Figure 2. Random road response.

As seen the proposed SMC provides the least body acceleration, thus achieving the most comfortable alternative. Also, it provides the least active force +satisfactory suspension stroke (<8 cm)

B. Bump road response

In [18], the system reaction resulting from a road bump is investigated with the example of an isolated bump in an otherwise smooth road surface. The equivalent ground displacement in this case is given by

$$x_r = \begin{cases} \frac{a}{2}\left(1-\cos\left(\frac{2\pi v}{l}t\right)\right), 0 < t < \frac{l}{v}; \\ 0, t > l/v \end{cases} \quad (13)$$

where v is the forward velocity of the vehicle and a and l, and h are the bump's height and length. The selected parameters are v = 45 km/h, l = 5 m, and a = 0.1 m. Fig. 3 displays the bump response at nominal load, including the acceleration of the body, suspension stroke (m), and active force (kN).

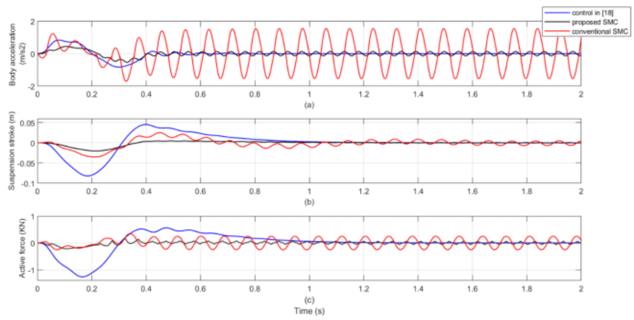

Figure 3. Bump road response at nominal load (M=320 kg): invariant-set control [18], conventional SMC, vs proposed SMC.

As seen the control objectives are satisfied: the vibration is damped in <1sec, the body acceleration is small, the suspension stroke is < 8 cm , and the active force is < 1.5 KN. It is evident that the proposed SMC is superior than the others. So, we limit our comparative study between the conventional SMC and the proposed one, and testing them at extremse conditions (heavy and light loads), Figs 4 and 5.

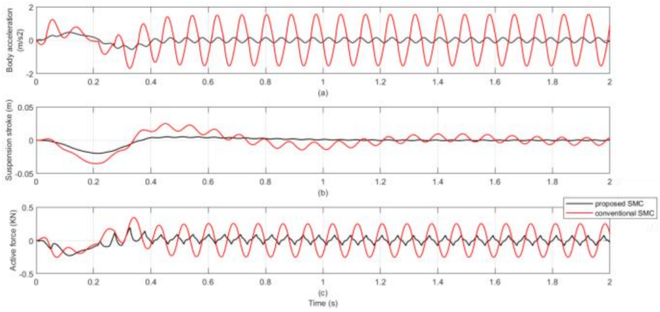

Figure 4. Bump road response at heavy load (M=390 kg): conventional SMC, vs proposed SMC.

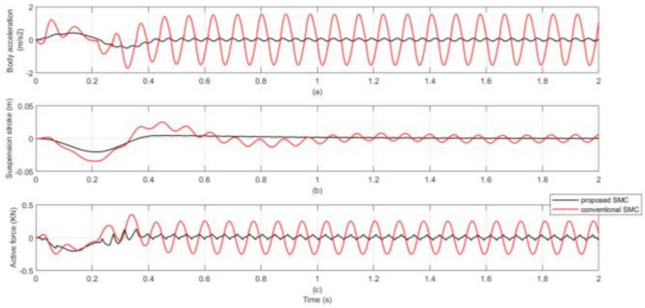

Figure 5. Bump road response at light load (M=250 kg): conventional SMC, vs proposed SMC

As seen, the proposed SMC is superior to the conventional one.

5 CONCLUSIONS

This research proposes a unique control approach for a suspension system of quarter cars. The SMC is the foundation of the design, and the invariant-ellipsoid approach is used to lessen the impact of mismatched road disturbances. Two common road profiles are used as simulations to demonstrate the efficacy of the suggested control mechanism.

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