# KRYLOV SUBSPACE MODEL ORDER REDUCTION OF LARGE SCALE FINITE ELEMENT DYNAMICAL SYSTEMS

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*The purpose of model order reduction (MOR) techniques is to provide a fast response approximate solution of a given system. Naturally the error induced due the approximation should be as low as possible. The paper presents the benefits of using a Krylov subspace based MOR for solution of large scale finite element (FE) dynamical systems. The main advantage of using a Krylov MOR is significantly shortening of the computational time while an induced error is almost negligible or very low. The process will be presented on structural harmonic analysis oflarge machine tool and a transient thermal analysis of thermal subsea equipment.*

**Keywords**
*model order reduction, Krylov subspace, large FEA, machine tool, subsea equipment, system dynamics, transient thermal analysis*

## 1. Introduction

The solution of FE problems usually involves solution large set of sparse algebraic equations. The number of unknowns in ordinary engineering FE model nowadays is commonly $10^6$ – $10^7$. The computational time needed to solve static analysis of such problems on current hardware is acceptable low (~hours). The issue arises when the harmonic or transient analysis is required. The solution then requires 10s or 100s of iterations and therefore requires days or even weeks to solve. Such long solution time effectively hinders this kind of simulations in common engineering practice. The MOR methods are one of possible ways to speed up the solution and make it usable in practice.

The idea of reduction of the number of unknowns during the solution of FE model is almost as old as FE method itself. The first MOR method was static reduction proposed by Guyan [Guyan 1965] and Irons [Irons 1965]. This method was introduced for structural mechanics problems but it is also valid for thermal analysis and other analyses regardless of the underlying physics. However this method is of questionable quality when using it for dynamic analyses as was shown in [Bushard 1981] and [Koutsovasilis 2008].

To remedy the insufficiencies in the static condensation method the component mode synthesis (CMS, [Bampton1968]) was proposed by Craig and Bampton. The CMS has become widely used by the engineering community. The CMS was used to efficiently conduct large-scale structural eigenanalysis [Aoyama 2001], but also for transient heat conduction analysis [Botto 2002] and heat conduction/convection analysis [Botto 2007]. Another field of application of the CMS is coupled physics simulations. The weakly coupled thermo-mechanical models were studied in [Nachtergaele 2010]. There is still active research regarding improvement of the CMS [Bai 2006].

Another method enhanced to approximate dynamic systems well is the improved reduced system (IRS) proposed by O'Callahan in [O'Callahan 1989]. Later, Friswell developed iterated version of IRS in [Friswell 1995]. The static condensation, CMS and IRS can be viewed as engineering approaches to reduce the number of equations.

The global error bounds and the preservation of passivity and stability are important questions posed on the MOR methods in a more mathematical point of view. One of the MOR techniques proposed in accordance to these questions is Krylov subspace reduction [Antoulas 2005] and Balanced truncation [Antoulas 2005]. Balanced truncation methods [Antoulas 2001] have a great advantage because there exists a priori global error bound. But it also has a great disadvantage in that the Lyapunov equation [Stykel 2002] needs to be solved in order to reduce the system. Thus the usage of Balanced truncation in reduction of large-scale systems is limited.

The Krylov subspace methods ([Antoulas 2005], [Grimme 1997b], [Grimme 1997a], [Bai, 2002]) are very interesting because of their iterative nature which allows reduction of large scale problems. The computational efficiency of Krylov subspace base MOR has encouraged wide interest in the method and therefore wide knowledge in different fields is available. We will mention most important observations to date. The passivity and stability preservation has been achieved using the Krylov MOR methods in [Odabasioglu 1997] and [Antoulas 2005]. A Krylov algorithm preserving structure of second order ordinary differential equations has been presented in [Bai 2005]. Handling of nonlinear convection coefficient was studied in [Feng 2005]. Reduction of coupled physic problems was studied in [Zukowski 2006] for the case of a thermo-mechanical model of packages and in [Puri 2009] for the case of structural-acoustic coupled models. Krylov subspace MOR was also successfully used in optimization of MEMS devices [Han 2005] and sensitivity analysis of structural frequency response [Han 2012]. One of the most important directions in development on the Krylov base reductions is a parametric model order reduction (PMOR). The PMOR allows preservation of parameters which the system depends on( [Baur 2011], [Sindler 2013] ). The dependence of parameters may be either linear or nonlinear.

The comparison of different model order reduction methods has been discussed in [Koutsovasilis 2008] where the Krylov subspace MOR method was found to be one of the best methods. The comparison of Krylov, CMS and Balanced truncation can be found in [Witteveen 2012].

Base on the findings presented in previous works the Krylov subspace based MOR is very robust and computationally efficient. The purpose of this study is to compare the usual solution methods and the Krylov subspace MOR in case of industry scale FE models. The Krylov subspace MOR will be assessed using the following requirements:

1. Low error in approximation
2. Fast computation

A low error in the approximated system is required as the objective is to replace results obtained by conventional methods with results obtained with the Krylov MOR. Reducing the computational time is the key to practical usage of any computational algorithm.

The article is organized in the following way: section 1 contains the introduction and the motivation of work; the models setup and problem description is considered in section 2; section 3 contains the description of Krylov subspace reduction; section 4 contains comparison of the results obtained by the methods and section 6 contains the conclusion and suggestions on future work.

## 2. The cases

The performance of the Krylov MOR will be studied on the two cases. The first one is harmonic analysis of machine tool and the second one is transient thermal analysis of subsea equipment.

The both cases are from different fields where specific requirements on system design arise. The goal of study is to assess the Krylov

MOR. Therefore only basic description of the cases is provided as they are not the ultimate goal of study.

### 2.1 The harmonic simulation of machine tool

The purpose of harmonic simulation of machine tool is to obtain frequency response function (FRF) on the tip of the tool. The obtained FRF could be used to simulate machining stability [Altintas 2000].

The FE model (Fig. 1) consists of the volume, shell, spring, matrix and mass elements [Sulitka 2012]. The mesh consists of about 1.5e6 nodes. The total degrees of freedom number is almost 6e6. The model was built in ANSYS v14.5.

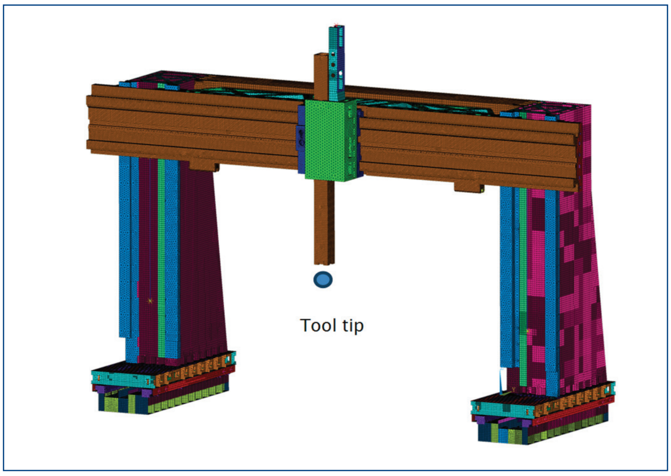


***Figure 1.*** *Machine tool FE model*

The underlying physics of the problem is described by a system of linear second order ordinary differential equations with constant coefficients

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = Fu(t) \quad (1)$$
$$y(t) = L^T x(t)$$

$$x(0) = x_0, \dot{x}(0) = x_1, \quad (2)$$

where $x(t) \in R^N$ is displacement vector of state variables, $u(t) \in R^N$ is input function and $y(t) \in R^m$ is output function. The matrices $M, C, K \in R^{NxN}$ are mass matrix, damping matrix and stiffness matrix. $F \in R^{Nxl}$, $L \in R^{Nxm}$ are input and output matrices. $x_0$, $x_1 \in R^N$ are initial conditions. The system matrices $M, C, K$ are constant therefore the system (1) linear time invariant system (LTI).

### 2.2 The transient thermal analysis of subsea equipment

The transient thermal analysis of subsea equipment is performed in order to assess insulation of equipment. The goal is to insulate the equipment enough to prevent the falling of the temperature under the hydrate formation temperature.

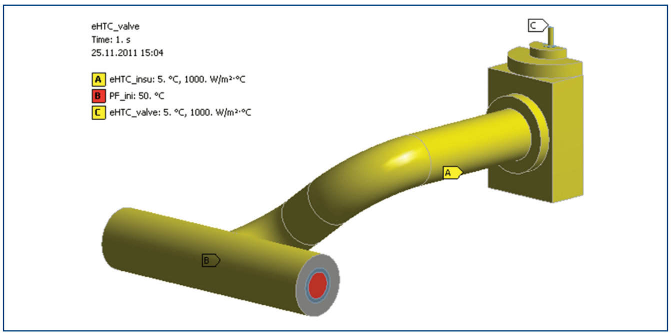


***Figure 2.*** *Subsea equipment FE model*

The FE model consists of the volume elements. The mesh consists of about 1e6 nodes. The number of DOFs is the same as number of nodes, 1e6. The model was built in ANSYS v14.0.

The physics of the problem is described by a system of linear first order ordinary differential equations with constant coefficients

$$C\dot{T}(t) + KT(t) = Qu(t) \quad (3)$$
$$y(t) = L^T T(t),$$

$$T(0) = T_0, \quad (4)$$

where $C \in R^{NxN}$ and $K \in R^{NxN}$ are specific heat matrix and conductivity matrix, $N$ is the dimension of the system. $Q \in R^{Nxl}$ is the input vector and $L \in R^{Nxm}$ is the output vector. $T \in R^N$ is the state vector (temperatures). $y(t) \in R$ is the output function and $u(t) \in R$ is the input function. In this case $u(t) = 1.T_0 \in R^N$ is initial conditions.

## 3. The Krylov subspace model order reduction

In this section only the basics behind Krylov reductions will be described. The reader is encouraged to read an excellent mathematical description of Krylov based reductions in [Antoulas 2005]. An overview of reduction methods is given in [Antoulas 2001]. Although optimal Krylov based reduction algorithms are available [Gugercin 2008] a simpler and possibly more computational efficient method will be used in this work – a block rational Krylov method [Elfadel 1997]. The structure of second order ODEs in (1) will be preserved using Bai's algorithm [Bai 2005].

### 3.1 The treatment of initial conditions

Let's consider using following coordinate transformation in case of eq. (1)

$$x(t) = \tilde{x}(t) + x_0,$$
$$\dot{x}(t) = \dot{\tilde{x}}(t) + x_1, \quad (5)$$
$$\ddot{x}(t) = \ddot{\tilde{x}}(t).$$

Substituting (5) into (1)

$$M\ddot{\tilde{x}}(t) + C\dot{\tilde{x}}(t) + K\tilde{x}(t) = Fu(t) - Cx_1 - Kx_0, \quad (6)$$

Right hand side of eq. (6) is enriched with constant term $-Cx_1 - Kx_0$ and we may consider this term as additional force. Then we may assume without loss of generality (1) with initial conditions in form of

$$x(0) = 0, \dot{x}(0) = 0. \quad (7)$$

Similarly let's consider using following coordinate transformation in case of eq. (3)

$$T(t) = \tilde{T}(t) + T_0,$$
$$\dot{T}(t) = \dot{\tilde{T}}(t). \quad (8)$$

Substituting (8) into (3) we get

$$C\dot{\tilde{T}} + K\tilde{T} = Qu(t) - KT_0$$
$$\tilde{T}(0) = 0, \quad (9)$$

therefore we can only consider zero initial conditions (IC) because using transformation (5) the nonzero IC is moved to the right hand side (RHS) of (9). This kind of treatment of a nonzero IC was introduced in [Feng 2004].

### 3.2 Krylov subspace based MOR

First we will consider the MOR of system (3). The Laplace transform of eq. (3) has the form

$$H(s) = L^T(sC + K)^{-1}Q. \tag{10}$$

H(s) is transfer function of system (3). The MacLaurin series of (10) has following form

$$H(s) = \sum_{i=0}^{\infty} m_l s^l, \tag{11}$$

where $m_l$ are the so-called moments of the transfer function:

$$m_l = L^T r_l, \tag{12}$$

where

$$r_0 = K^{-1}Q$$

$$r_1 = K^{-1}Cr_0$$

$$r_l = K^{-1}Cr_0.$$

The first *n*-vectors $r_l$ span Krylov space

$$K_n = span(r_0, \cdots, r_{n-1}). \tag{13}$$

Let $V_n$ be the orthonormal basis of $K_n$

$$K_n = span(V_n), \quad V_n^T V_n = I, \quad V_n \in R^{Nxn}. \tag{14}$$

The projection of state coordinates *T* onto $K_n$ using $V_n$ is called generalized state coordinates $q \in R^n$

$$T = V_n q + \epsilon. \tag{15}$$

The error $\in\ \in R^n$ in the projection rises while performing projection of x onto $K_n$ (Figure 3).

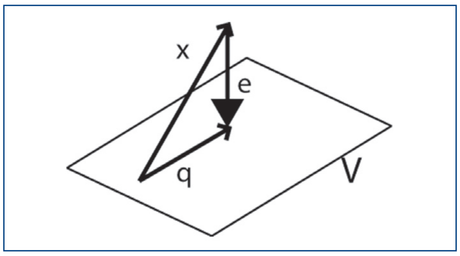


***Figure 3.*** *Projection onto Kn*

The system equation (3) in generalized coordinates has the form

$$CV_n\dot{q} + KV_n q = Q. \tag{16}$$

Using Galerkin method and thus imposing residual forces *R* to be perpendicular to subspace $K_n$

$$CV_n\dot{q} + KV_n q - Q = \mathrm{R} \perp \mathrm{K_n}. \tag{17}$$

The vector of residual forces *R* arises due to the error in the approximation $\in$ in (15). Forcing the Galerkin conditions leads to the reduced system of equations

$$C_n\dot{q} + K_n q = Q_n$$
$$\tilde{y}(t) = L_n^T q(t), \tag{18}$$

where

$$K_n = V_n^T K V_n$$
$$C_n = V_n^T C V_n$$
$$Q_n = V_n^T Q$$
$$L_n = \mathrm{V_n L}, \tag{19}$$

where

$$C_n, K_n \in R^{nxn}, Q_n \in R^n, L_n \in R^{nxm}.$$

The transfer function of the reduced system (18) has the form

$$H_n(s) = L_n^T(sC_n + K_n)Q_n. \tag{20}$$

The above procedure assures that the first *n* moments of the transfer function (10) of the full system equals to the first *n* moments of the transfer function (20) of the reduced system [Antoulas 2005].

The procedure for the second order ODEs in eq. (1) is similar. The Laplace transform of eq.(1) has form of

$$H(s) = L^T(s^2M + sC + K)^{-1}F. \tag{21}$$

And the McLaurin series of transfer function (21) has the form

$$H(s) = \sum_{i=0}^{\infty} m_l s^l, \tag{22}$$

where $m_l$ are the so-called moments of the transfer function

$$m_l = L^T r_l, \tag{23}$$

$$r_0 = K^{-1}F$$
$$r_1 = -K^{-1}Dr_0$$
$$r_l = -K^{-1}(Dr_{l-1} - Mr_{l-2}). \tag{24}$$

The first *n* vectors $r_l$ span Krylov space $K_n$ with orthonormal basis $V_n$. The definition of generalize coordinate *q* is similarly

$$x = Vq + \epsilon. \tag{25}$$

We obtain reduced system of eq. (1) substituting generalized coordinates *q* into eq. (1) and using Galerkin method. The reduced equations have the form of

$$M_n\ddot{q}(t) + C_n\dot{q}(t) + K_n q(t) = F_n u(t)$$
$$\tilde{y}(t) = L_n^T q(t), \tag{26}$$

where

$$M_n = V_n^T M V_n$$
$$K_n = V_n^T K V_n$$
$$C_n = V_n^T C V_n$$
$$F_n = V_n^T F. \tag{27}$$

Where

$$M_n, C_n, K_n \in R^{nxn}, F \in R^n, L_n \in R^{nxm}.$$

The error induced by the projection (15) in the output function *y(t)* has the form

$$\epsilon = \max_{t>0} |y(t) - \tilde{y}\,(t)|. \tag{28}$$

An a priori expression for error norm (28) is not known although there exist algorithms minimizing the error ([Gugercin 2008], [Flagg 2011]). The algorithm used in this paper to produce the reduced order systems is the block Arnoldi algorithm [Elfadel 1997].

There exist wide possibilities to improve computational performance of Krylov methods. One of most obvious options is parallelization [Yetkin 2009]. Another is to use an iterative algorithm to solve the system [Beattie 2006]. The presented case is of medium size and it is therefore suitable to use the direct sparse solver [Davis 2006].

The procedure is easily extended to a multi-input/multi-output case where $Q$, $F \in R^{N\times l}$ and $L \in R^{N\times m}$. The size of the reduced system is determined by the size of $Q$ and $L$. However it is possible to use the superposition property [Benner 2008] to keep the matrices small.

## 4. Results

The following case studies compare computational efficiency of solvers in ANSYS software package and efficiency of Krylov MOR. To assess quality of results obtained using MOR the error norm is evaluated.

### 4.1 The harmonic simulation of machine tool

The frequency response function at the tip of the tool has been computed using full FEA model and Krylov subspace MOR with first n=100 moments. Additionally FRF computed using modal reduction is provided for comparison. The comparison is shown in *Tab. 1*. The error norm is concluded to be

$$\epsilon = \max_{f\in(0,400)} |y(f) - \tilde{y}(f)| \cong 4e-6. \qquad (29)$$

| | MOR | FRF simulation | Total time |
|---|---|---|---|
| Full harmonic | – | 111 hours | 111 hours |
| Modal reduction | 40 min | < 1 s | 40 min |
| Krylov MOR | 6,9 min | < 1 s | 6,9 min |

***Table 1.*** *Comparison of computational times*

The *Fig. 4* and *Fig. 5* show FRFs in axis X and Y. The results clearly show the quality of approximation obtained using Krylov MOR as well as computational efficiency compared to full solution and modal reduction.

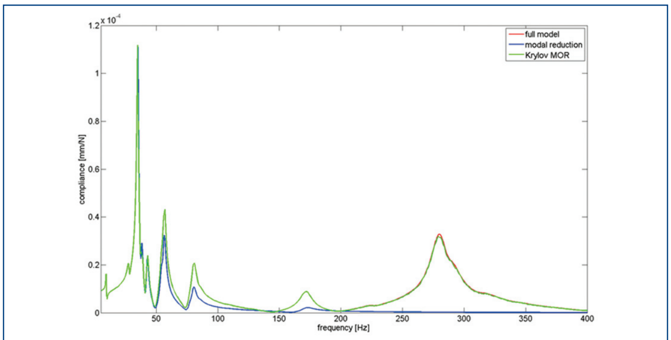


***Figure 4.*** *Comparison of FRF, axis X*

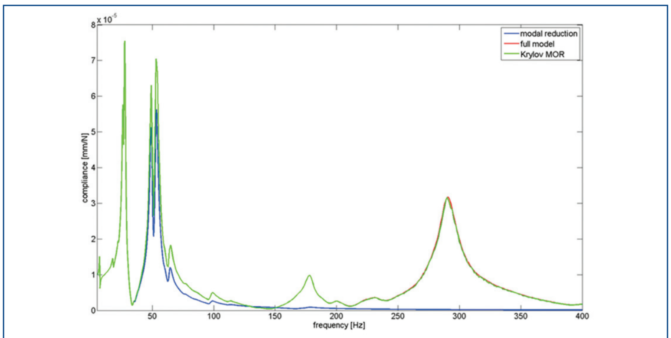


***Figure 5.*** *Comparison of FRF, axis Y*

The Krylov MOR is about 1000x more computational efficient than full solution and the approximation error is almost negligible.

### 4.2 The transient thermal analysis of subsea equipment

The transient thermal analysis of the subsea equipment using different solvers and the time stepping strategies has been thoroughly studied in [Sindler 2013]. The transient thermal analysis has been run for time=54000s using incomplete Cholesky conjugate gradient (iccg) solver [Saad 2003] with maximum time step 54s and 540s and using Krylov MOR with time step 1s. The error norm for each time is given as

$$\varepsilon(t) = \max\big(T(t) - T_{54}(t)\big), \qquad (30)$$

where $T_{54}(t)$ is result obtained using iccg solver with maximum time step 54s. This result is considered converged in time domain. The Figure 6 shows the resulting error norm for the result obtained using Krylov MOR with n=100 ($\varepsilon(Tr_{100})$) and the results obtained using iccg solver with maximum time step 540s. The result clearly shows that Krylov MOR produces lesser error than iccg solver with maximum time step 540s.

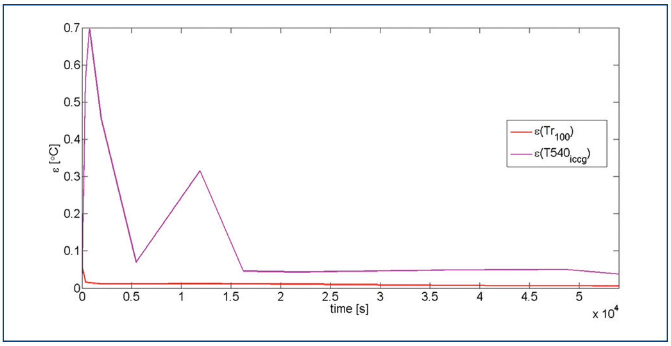


***Figure 6.*** *The error norm*

The *Tab. 2* shows comparison of computational times. The result again shows that Krylov MOR is superior to standard solver –iccg. The time step strategy with maximum time step = (1/100 * simulation time) = 540s is highest fidelity simulations used in industry. The results obtained using this time stepping is not fully converged in time domain (Fig. 6) and it is possible to get more precise results using Krylov MOR than using this time stepping strategy. Moreover computational time using Krylov MOR is about 70x more efficient (Tab. 2).

| | MOR | Cool down simulation |
|---|---|---|
| iccg maximum time step 54 s | – | 59 hours |
| iccg maximum time step 540 s | – | 6,4 hours |
| Krylov MOR n=100 | 319 s | <1 s |

***Table 2.*** *Comparison of computational times*

## 5. Conclusions

The performance of the Krylov subspace MOR has been studied on the two different cases. It has been found in both cases that Krylov subspace MOR is superior to standard solvers in terms of computational time. It has also been concluded that approximate results obtained using Krylov MOR are excellent. The error in the results is almost negligible.The requirements posed in section 1 are met and therefore the conclusion is that Krylov MOR is very suitable replacement for standard solvers for transient and harmonic analysis.

The future work should explore possible implementation of substructuring similar to substructuring in static reduction [Guyan 1965] and CMS [Bampton 1968]. The substructuring could allow even larger model to be reduced in very short time and also it would allow multibody assemblies of Krylov based reduced components.


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