# A MODAL PARAMETER APPROACH FOR RECEPTANCE COUPLING OF TOOLS

# C. BRECHER, P. CHAVAN, M. FEY, M. DANIELS

Laboratory for Machine Tools and Production Engineering (WZL) of RWTH Aachen University,
Aachen, Germany

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e-mail: p.chavan@wzl.rwth-aachen.de

Receptance coupling methods for predicting tool point dynamics often use an adapter or calibration gauge for extracting the spindle-tool interface dynamics. This, however, necessitates the measurement of the rotation to force and rotation to moment receptances at the free end of the clamped adapter which is difficult to measure practically. The present article provides a new method for determining the rotation to moment receptance as part of the Receptance Coupling Substructure Analysis (RCSA). The formulations in the current literature, suffer from several disadvantages such as having to use several calibration gauges, complex mathematical formulations, multiple measurements of direct and cross receptances, etc. The proposed approach overcomes these disadvantages by providing a simpler formulation with only three measured receptance on a single adapter. A modal parameter approach is used which shows better prediction of the rotation to moment receptance and satisfactory conformity of the predicted TCP compliance with the measured compliance.

## **KEYWORDS**

receptance coupling, RCSA, modal parameters, milling tool, rotation to moment receptance

## 1. INTRODUCTION

Among the various factors influencing the machining accuracy of a Machine tool, the dynamic behaviour relative between the workpiece and tool plays a crucial role [Weck 2006]. Not only does the dynamic behaviour determine the achievable surface roughness, but also the maximum cutting depth of a machine without 'chatter'. All components lying within the force flux of process forces, for example the cutting tool, workpiece, tool holder, spindle, feed drive etc., influence the dynamic behaviour. A variation in the dynamic property of any individual component alters that of the machine tool. The cutting tool, being the part which is changed most often and having varying dimensions, significantly influences this behaviour. An accurate quantitative prediction about the modified dynamics of the machine tool due to tool change would be beneficial in making appropriate changes in process parameters for avoiding chatter.

Receptance Coupling Substructure Analysis (RCSA) has been widely used for predicting the Tool Center Point (TCP) receptance after assembly with a tool [Schmitz 2002, Park 2005, Schmitz 2003, Schmitz 2005]. Previous approaches to RCSA considered only translation dynamics of the spindle-tool interface and ignored the rotational DOF of the joint [Schmitz 2002]. Later, Park et al. showed that the consideration of rotation to force and rotation to moment compliances resulted in significant improvement of the prediction [Park 2003]. However, measurement of these compliances or receptances is difficult and non trivial. There are numerous approaches available in literature which

estimate the remaining components of the compliance matrix at the spindle-tool interface using only the displacement to force compliance. Most approaches rely on the measurement with a simple cylindrical adapter attached to the spindle. For example, Park et al. propose the use of short and long cylindrical calibration gauges for measuring direct and indirect displacement to force receptances and then substituting in the proposed mathematical formulation to get the remaining tool holder and joint receptances. Due to the complicated mathematical formulation for calculating the joint receptances, this method has not found wide implementation. Schmitz and Duncan in [Schmitz 2005] propose measuring FRFs at three points on the adapter and using the second order backward finite difference method for determining the rotation to force compliances and rotation to moment compliances at the free end of the assembled adapter. Inverse receptance coupling is then used to decouple the adapter and find the interface receptance matrix. Although this method is relatively easy to implement, it lacks in the accuracy of prediction and also the error in prediction of rotation to moment receptance.

In order to overcome these disadvantages, Albertelli et al. proposed a formulation for calculating the full matrix receptance of the adapter using the finite difference methodology [Albertelli 2013]. Although this method delivers satisfactory prediction of the rotation to moment receptance, it requires the measurement of additional six direct and cross receptances for the prediction. The large number of measurement data involved in the calculation also makes the result susceptible to measurement noise.

To improve on the methods available in the literature, this paper proposes a new formulation for determining the rotation to moment compliance of the free end of an assembled adapter using a modal parameter formulation. The proposed method retains the use of second order backward finite difference approach for determining non diagonal terms of the receptance matrix.

The following section describes the proposed methodology along with the mathematical formulation for determining rotation to torque compliance. This is subsequently validated by a simulation of the inverse receptance coupling of an adapter. Finally, an experimental validation of the approach is described. The paper is then concluded with a short summary.

# 2. RSCA FOR TOOL COUPLING

The general approach for predicting TCP compliance of an assembled tool along with the proposed changes to this method is explained with the help of Fig. 1. This method retains the advantages of using a cylindrical adapter for extracting the interface compliance matrix. The adapter (Part A) is clamped with the spindle and the receptance matrix at the free end,  $G_{33}$  is to be dertermined. The direct displacement to force receptance at interface 3,  $H_{33}$  can be measured using an absolute excitation, for example with an impulse hammer and acceleration sensor. The rotation to force receptance,  $L_{33}$  and displacement to moment receptance,  $N_{33}$  are estimated using the backward finite difference method proposed in [Schmitz 2005],

$$L_{33} = \frac{{}_{3}H_{33} - 4H_{32} + H_{31}}{2S} \tag{1}$$

For this, two cross-receptances  $H_{32}$  and  $H_{31}$  are measured at intervals of S as shown in Fig. 1a). For linear structures, rotation to force and displacement to moment receptances can be assumed to be identical. The mathematical formulation for the estimation of rotation to moment receptances,  $P_{33}$  introduced in the next sub-section. The matrix of receptences at the free end '3' of the clamped adapter is given by,

$$G_{33} = \begin{bmatrix} \frac{X_3}{F_3} & \frac{\Theta_3}{F_3} \\ \frac{X_3}{M_3} & \frac{\Theta_3}{M_3} \end{bmatrix} = \begin{bmatrix} H_{33} & N_{33} \\ L_{33} & P_{33} \end{bmatrix} = G_{A,33} - G_{A,35a} (G_{A,5a5a} + G_{55})^{-1} G_{A,5a3}$$
(2)

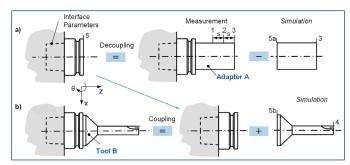


Figure 1. a) Decoupling of adapter b) coupling of simulated model of tool

All terms on the right hand side of equation (2), except  $G_{55'}$  are the receptances of the freely hanging (unconstrained) adapter, denoted by the subscript 'A'. In this paper an Euler-Bernoulli beam model is used to calculate these receptances. The remaining receptance matrix the spindle-tool interface can then be expressed as,

$$G_{55} = \begin{bmatrix} H_{55} & N_{55} \\ L_{55} & P_{55} \end{bmatrix} = G_{A,35a} (G_{A,33} - G_{33})^{-1} G_{A,5a3} - G_{A,5a5a}$$
(3)

The derived interface receptance matrix can then be utilized for coupling any tool 'B' with identical interface type. The TCP FRF prediction for the new tool, is given by,

$$G_{44} = \begin{bmatrix} H_{44} & N_{44} \\ L_{44} & P_{44} \end{bmatrix} = G_{B,44} - G_{B,45b} (G_{B,5b5b} + G_{55})^{-1} G_{B,5b4}$$
(4)

## 2.1 Mathematical formulation and simulation

The derivation of the adapter receptance matrix occurs in 3 steps. Firstly, the direct displacement to force compliance and cross compliances are measured using, for example, an impact hammer. Secondly, the rotation to force receptance,  $L_{33}$  and displacement to moment receptance,  $N_{33}$  are derived using the finite difference method in equation 1. Now, the receptances  $H_{33}$  and  $L_{33}$  correspond to the direct compliances of the same measurement point '3' when excited or measured in two different directions (translation and rotation). A modal parameter estimation of the two FRFs is subsequently carried out which gives a pole vector  $\boldsymbol{p}$  of size  $1\times n$  modal matrix,  $\boldsymbol{\varphi}$  of size  $2\times n$ , where n is the number of poles considered.

This matrix contains the modal parameters of the structure at point '3' for both, the translational degree of freedom (DoF) from  $H_{33}$  as well as rotational DoF from  $L_{33}$ . A receptance for excitation and displacement in the rotational direction can hence be found out using modal superposition such that,

$$P_{33} = \sum_{k=1}^{n} \left( \frac{\varphi_{jk} \varphi_{jk}}{i\omega - p_k} + \frac{conj(\varphi_{jk} \varphi_{jk})}{i\omega - conj(p_k)} \right) \tag{5}$$

where, j represents the row corresponding to the modal parameters of the rotational DoF. The following subsection shows the implementation of the above strategy for a simulated spindle and adapter. In order to evaluate the performance of the above formulation, a simplified spindle-tool assembly with and without the adapter was created in an FE-environment (Fig. 2a). A decoupling of the adapter in X direction is to be achieved. Three condensation nodes were created on the adapter at a distance of 25 mm and translational FRFs were simulated. The receptance and cross-receptances were used to calculate  $L_{33}$  (equation 1).  $P_{33}$  was subsequently calculated from the estimated modal parameters as in equation 5. Fig. 2b) shows a comparison of rotation to moment receptances from the formulation in [Schmitz 2005] with the proposed approach.

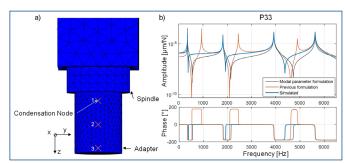


Figure 2. a) Simple FEM model for spindle-adapter  $\mathbf{b}$ ) predicted and simulated receptance P33

As can be seen, the proposed formulation provides a much better estimate of the  $P_{\rm 33}$  receptance. Since the structure is constrained at one end, the vibration modes of the structure in X direction will be observable in the receptance matrix  $G_{\rm 33}.$  In other words, all FRFS in the receptance matrix share the same poles. Unlike the formulation in [Schmitz 2005], the current approach ensures that the poles of the calculated  $P_{\rm 33}$  FRF remain common.

#### 3. EXPERIMENTAL VALIDATION

The following test case was considered for experimental validation of the proposed methodology. A cylindrical adapter of dimension 50 x 64 mm with an HSK 63A interface is to be decoupled from the spindle of a vertical milling machine. The tool to be coupled is a milling tool with two cutting edges, has the same interface and has a length of 125 mm from the interface (Fig.3a). A prediction of compliance at the TCP with the milling in X direction is to be realized. The displacement to force direct and cross receptances at the three points were measured by absolute excitation with an impulse hammer and a laser vibrometer for displacement measurement. The distance between the measurement points was taken to be, S=25 mm. The derived FRFs were then used to calculate the remaining receptances  $(L_{33}, N_{33}, P_{33})$  of the matrix of recptances at the free end of adapter as explained in Section 2. A polymax LSCF (Least Square Complex Frequency) algorithm was used for estimating the modal parameters of measured FRFs.

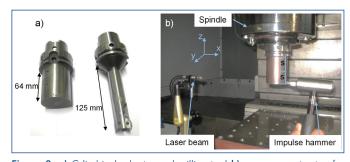


Figure 3. a) Cylindrical adapter and milling tool  ${\bf b}$ ) measurement setup for receptance measurement

For modelling the free-free dynamics of the tool and the adapter, the closed form expression of the Euler-Bernoulli beam was used [Bishop 1955]. For modelling the conical curvature of the milling tool using Euler-Bernoulli expression, it was radially dissected into five smaller discs. Receptances of each disk were coupled with the adjacent disk to form the conical part and was subsequently coupled with a cylinder of dimension corresponding to the cylindrical part of the tool. Alternative to modelling the free-free dynamics using a continuous beam model, an FE-model can also be used for simulating the free-free FRFs of the parts. Finally, solving equation 4 for the tool to be coupled gives the receptance matrix of the free end of the coupled tool,  $G_{44}$ . Where, the component  $H_{44}$  represents the displacement to force receptance

and is plotted in Fig. 4. As can be seen, the prediction agrees well with the measured FRF throughout the measured frequency band. The coherence of the measurement and prediction above 2.5 kHz was below 0.5 and is hence not included. The modes below 500 Hz correspond to the modes of the machine and interface and those a 1240 Hz and 1360 Hz correspond to the bending modes of the tool. Fig 4. also shows the simulated tool-tip FRF of the milling tool when attached to a perfectly rigid structure. This shows the importance of considering and calculating receptance matrix of the spindle-tool interface  $G_{55}$  using inverse receptance coupling (Eq. 5).

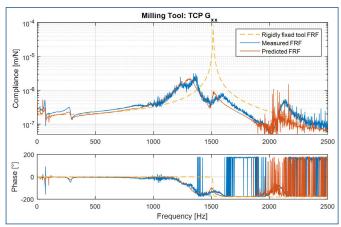


Figure 4. Comparison of TCP FRF in X direction for measured and predicted FRFs

### 4. CONCLUSION

The measurement of rotation to moment receptances at the free end of a clamped adapter is not practical. For this, there are several solutions available in literature where the receptance matrix is predicted from only direct displacement to force compliance. However, these methods suffer from the disadvantage of either a complex mathematical formulation or require the measurement of several additional direct and cross receptances making the results further sensitive to measurement noise. This paper presents a new methodology for predicting the rotation to moment receptances using a relatively simple modal parameter formulation and without conducting additional measurements.

The results of the experimental validation show a good conformity with the measured FRF. The assumption of a rigid connection between the interface and tool seem to be valid for the given test case where the milling tool is clamped with the spindle. However, for the case where a tool holder is used, better results can be expected when the tool holdertool connection is attributed a translational and rotational stiffness and damping [Schmitz 2005]. An Euler Bernoulli beam model delivers satisfactory results for the modelling the adapter and tool despite the relatively large d/l ratios of the beam and beam sections.

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# **CONTACTS:**

Prateek Chavan, M.Sc.

Laboratory for Machine Tools and Production Engineering (WZL) **RWTH Aachen University** 

Steinbachstraße 19, 52074 Aachen, Germany

e-mail: p.chavan@wzl.rwth-aachen.de